

**Please Note:** (E) NOTA denotes none of the above (answers are correct).

- 1 A unit circle is cut into 2012 pieces (all equal in arc length) by 2012 distinct points. Three of them are chosen at random. The probability that these three points form a right triangle is  $m/n$  where  $m$  and  $n$  are relatively prime integers. What is  $m + n$ ?

(A) 673491      (B) 673686      (C) 2014      (D) 2013      (E) NOTA

- 2 Consider the equation of circle  $\omega$  given by:  $(x + 1)^2 + (y + 1)^2 = 1$ . Let  $u = ab$  where the point  $(a, b)$  lies on the circle. Find the sum of the largest and smallest value of  $u$ .

(A)  $3 + 2\sqrt{2}$       (B)  $\frac{3}{2} + \sqrt{2}$       (C)  $\sqrt{2}$       (D)  $\frac{3}{2}$       (E) NOTA

- 3 There are two distinct lines tangent to circle  $\omega$  (see question (3)) which pass through  $(5, 3)$ . The larger of the slopes of these two lines can be expressed in the form  $\frac{x + \sqrt{y}}{z}$  where  $x, y, z \in \mathbb{Z}$  and where  $y$  isn't divisible by the square of any prime. Find the sum of  $x, y$ , and  $z$ .

(A) 120      (B) 110      (C) 100      (D) 90      (E) NOTA

- 4 What is the area of the triangle formed by connecting the points  $(0, 0, 0)$ ,  $(1, 1, 2)$ , and  $(3, 4, 2)$ ?

(A)  $\sqrt{61}$       (B)  $\sqrt{53}$       (C)  $\sqrt{61}/2$       (D)  $\sqrt{53}/2$       (E) NOTA

- 5 The vectors  $\langle 1, 1 \rangle$  and  $\langle 7, k \rangle$  will be orthogonal for which value(s) of  $k$ ?

(A) 7      (B)  $-7$       (C) 7 and  $-7$       (D)  $-1$       (E) NOTA

- 6 What is the maximum number of regions 2011 lines divide a plane into (you may assume that all the lines lie in the given plane)?

(A)  $\frac{2011^2 + 2009}{2}$       (B)  $2011^2 + 2011$       (C)  $\frac{2011^2 + 2011}{2}$

(D)  $\frac{2011^2 + 2013}{2}$       (E) NOTA

- 7 Consider two integers,  $a$  and  $b$ , which are both randomly chosen from the interval  $(0, 2012)$ . What is the probability that the sum of the slopes of the asymptotes of the hyperbola shown below is zero?

$$\frac{(x - b)^2}{a^2} - \frac{y^2}{4b^2} = 1$$

(A)  $1/2011$       (B)  $1/2012$       (C)  $\frac{1}{\binom{2011}{2}}$       (D)  $\frac{1}{\binom{2010}{2}}$       (E) NOTA

- 8] The three points  $(1, 1)$ ,  $(k, 3)$ , and  $(2, k)$  will all lie on a circle except for  $t$  (an integer) values of  $k$ . Find the sum of the possible values of  $k$  and the value of  $t$ .

(A)  $4 + \sqrt{2}$       (B) 4      (C)  $2 + \sqrt{2}$       (D)  $1 + \sqrt{2}$       (E) NOTA

For Questions (9), and (10) let  $\vec{i} = \langle 1, 0, 0 \rangle$ ,  $\vec{j} = \langle 0, 1, 0 \rangle$ , and  $\vec{k} = \langle 0, 0, 1 \rangle$ .

- 9] Let  $\vec{u} = i \times (j \times k)$ . What is  $\|\vec{u}\|$  ?

(A) 0      (B) 1      (C)  $\sqrt{3}$       (D) 3      (E) NOTA

- 10] Let  $\vec{v} = i \times (j \times (k \times (i \times (j \times k \times (i \times (j \times (j \times k))))))$ . Find  $\|\vec{v}\|$ .

(A) 0      (B) 1      (C)  $\sqrt{3}$       (D) 3      (E) NOTA

- 11] Consider the two sets:  $A = \{(x, y) \mid x^2 + y^2 = 4\}$ , and  $B = \{(x, y) \mid (x - 3)^2 + (y - 4)^2 = r^2\}$ . If the intersection of sets  $A$  and  $B$  only has one element, what is the sum of the possible values of  $r$  (assuming they are positive)?

(A) 3      (B) 4      (C) 7      (D) 10      (E) NOTA

- 12] By definition, if  $\vec{a}$  and  $\vec{b}$  are vectors, and  $\theta$  is the angle between them, then  $\|\vec{a} \times \vec{b}\|$  is equivalent to . . .

(A)  $\sin \theta$       (B)  $\cos \theta$       (C)  $\|\vec{a}\| \|\vec{b}\| \cos \theta$       (D)  $\|\vec{a}\| \|\vec{b}\| \sin \theta$       (E) NOTA

- 13] Let  $\theta$  be the angle, in degrees, between the two vectors  $\langle 1, 1, 1 \rangle$  and  $\langle 2, 4, 3 \rangle$ . In which of the following intervals does  $\theta$  lie? **NOTE:** all intervals are in degrees.

(A)  $(0, 20)$       (B)  $(20, 40)$       (C)  $(40, 60)$       (D)  $(60, 80)$       (E) NOTA

- 14] Consider the plane which passes through the points  $(1, 2, 1)$ ,  $(0, 3, 1)$ , and  $(2, 5, 7)$ . What is the shortest distance between this plane and the point  $(-1, -2, 2)$ ?

(A)  $\frac{5\sqrt{11}}{11}$       (B)  $\frac{10\sqrt{11}}{11}$       (C)  $\frac{5\sqrt{22}}{11}$       (D)  $\frac{10\sqrt{22}}{11}$       (E) NOTA

- 15] Vectors,  $\vec{a}$  and  $\vec{b}$  are said to be orthogonal if . . . (in the following answer choices,  $\theta$  is the smaller angle between the two vectors).

(A)  $\sin \theta = 0$       (B)  $\cos \theta = 0$       (C)  $\tan \theta = 0$

(D)  $\tan \theta$  is undefined      (E) NOTA

- 16 Find the area of the polar graph:  $r = \frac{5}{3 + 2 \cos \theta}$ .
- (A)  $3\pi\sqrt{5}$       (B)  $3\pi\sqrt{6}$       (C)  $3\pi\sqrt{7}$       (D)  $6\pi\sqrt{2}$       (E) NOTA
- 17 What is the eccentricity of the graph of:  $2012! \cdot y^2 = \frac{1}{2011!}(x - 2010!)$ ?
- (A) 0      (B) 1      (C)  $2012! \cdot 2011!$       (D)  $\frac{1}{2011! \cdot 2012!}$       (E) NOTA
- 18 In a certain ellipse, one of its foci lies at  $(2012, 2012)$ . If the point  $(0, 0)$  is on the graph, and the ellipse has major axis length  $2012^2\sqrt{2}$ , then which of the following might be the coordinate of the other focus?
- (A)  $(2012, -2012)$       (B)  $(-2012, 2012)$       (C)  $(-2011^2 \cdot 2012^2, 0)$
- (D)  $(2011^2 \cdot 2012^2 \cdot 2, 0)$       (E) NOTA
- 19 What is the slope of the line tangent to the circle  $(x - 2)^2 + (y - 3)^2 = 2$  at the point with  $x$ -coordinate 2 and  $y$ -value greater than 3?
- (A)  $3 + \sqrt{2}$       (B)  $(3 + \sqrt{2})/2$       (C) 0      (D)  $3 - \sqrt{2}$       (E) NOTA
- 20 The polar point  $(2, \frac{\pi}{3})$  in rectangular coordinates is . . .
- (A)  $(1, \sqrt{3})$       (B)  $(\sqrt{3}, 1)$       (C)  $(2, 2\sqrt{3})$       (D)  $(2\sqrt{3}, 2)$       (E) NOTA
- 21 How many distinguishable permutations of the words *analytic* exist?
- (A) 40,320      (B) 20,160      (C) 10,080      (D) 5040      (E) NOTA
- 22 The points  $(1, 1)$ ,  $(5, 3)$ , and  $(a, b)$  form an equilateral triangle. Find the minimum value of  $ab$ .
- (A)  $-4 - 3\sqrt{3}$       (B)  $-4\sqrt{3}$       (C)  $3\sqrt{3} - 4$       (D) 1      (E) NOTA
- 23 What is the angle between the space diagonal of a cube and one of its face diagonals?
- (A)  $\sqrt{3}/2$       (B)  $\sqrt{6}/2$       (C)  $\arccos\left(\frac{\sqrt{6}}{2}\right)$       (D)  $\arccos\left(\frac{\sqrt{3}}{2}\right)$       (E) NOTA

- 24] Let  $R$  be the circumradius and  $r$  be the in-radius of a generic triangle  $ABC$ . The law of sines says that:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = \frac{1}{X}$$

Which of the following is  $X$ ?

- (A)  $R$       (B)  $2R$       (C)  $r$       (D)  $2r$       (E) NOTA
- 25] Consider the graph  $f$  defined parametrically by:  $x = \sin(2t)$  and  $y = \cos t$ . What kind of graph is this?
- (A) Parabola      (B) Circle      (C) Ellipse      (D) Hyperbola      (E) NOTA
- 26] The area of one of the “petals” on the graph of the rose  $r = 12 \cos(6\theta)$  is  $6\pi$ . What is the area of the entire graph?
- (A)  $12\pi$       (B)  $24\pi$       (C)  $36\pi$       (D)  $72\pi$       (E) NOTA
- 27] The circle which passes through the points  $(1, 1)$ ,  $(0, 2)$ , and  $(4, 0)$  can be expressed in the form  $x^2 + y^2 + Ax + By + C = 0$ . Compute  $ABC$ .
- (A)  $-1280$       (B)  $-128$       (C)  $128$       (D)  $1280$       (E) NOTA
- 28] The solutions to  $x^3 - 1 = 0$ , when graphed in the complex plane, form an equilateral triangle. Find this triangle’s area.
- (A)  $3\sqrt{3}/4$       (B)  $\sqrt{3}/4$       (C)  $3/4$       (D)  $\sqrt{3}$       (E) NOTA
- 29] The graph of  $\pm(x^2 + y^2)^{\frac{3}{2}} = 2xy$  is that of a . . .
- (A) *rose*      (B) *lemniscate*      (C) *cardioid*      (D) *spiral*      (E) NOTA
- 30] Suppose the centroid of triangle  $EFG$ , where the vertices have coordinates  $E(1, 1)$ ,  $F(2, 3)$ , and  $G(4, 7)$ , is  $C(m, n)$ . Find the sum  $m + n$ .
- (A)  $9$       (B)  $17/2$       (C)  $17/3$       (D)  $6$       (E) NOTA