**Alpha Analytic Geometry**
**Solutions**

1. Horizontal lines are in the form $y = number$, and the given $y$-coordinate is 2. The line is therefore $y=2$, **B**.

2. Using the distance formula $D=\frac{|Ax+By+Cz+D|}{\sqrt{A^2+B^2+C^2}}$, we have
$$D=\frac{|2(2)+2(-1)+(-1)(5)-3|}{\sqrt{2^2+2^2+1^2}}=\frac{|-6|}{3}=2, \mathbf{C}.$$

3. The line will have slope $m_{\perp}=-\frac{11-9}{4-7}=\frac{2}{3}$. The point-slope form is given by **D.**

4. $x^2+1=2x+16 \to x^2-2x-15=0 \to (x-5)(x+3)=0$. The points of intersection are $(5,26)$ and $(-3,10)$, the sum of whose coordinates are 38, **B**.

5. Replacing $x$ with $-x$ and $y$ with $-y$ is the only way to produce the original expression. This is the test for origin symmetry, **C**.

6. Opposite angles in an inscribed quadrilateral are always supplementary. Since the angles must have a measure of less than 180 agrees, one angle will be a Quadrant I angle and the other a Quadrant II angle. The cosine values of these angles will be opposites of one another, so their sum will be 0. The sine values of the angles will be the same. There are two true statements given, so the answer is **B.**

7. The equation $x^2+y^2+8y-2x-20=0$ factors into $(x-1)^2+(y+4)^2=20+1+16=37$, so the area is $37\pi$, **D**.

8. $f(x)=\frac{x^3-2x^2-29x-42}{x^2-9}=\frac{(x+3)(x-7)(x+2)}{(x+3)(x-3)}$, so there is a hole in the graph at $x=-3$. A hole is a removable discontinuity, **A.**

9. Point $N$ could be located at either $(-2,-8)$ or $(10,-8)$. However, the former point does not yield a parallelogram, if we strictly go by the point ordering of SAND. So the only possible solution is (10, -8), or c = 10. **E.**

10. $f(x)$ has solutions at $x=-2$ and $x=6$, so $2-2x$ must be equal to $-2$ and 6. This occurs when $x=\pm 2$, **A.**

    Also, $y=-3f(2-2x)=-3f\left[-2(x-1)\right]$. Using patterns of transformations, we know that the negative sign forces a $y$-axis reflection, so the intercepts become 2 and –6.

The 2 cuts the intercepts in half, and the –1 moves the points to the right. This puts the points at $x=\pm 2$, **A.**

11. Odd-theta gives that number of petals; even-theta gives twice as many petals. We have 6(24) + 3(15) + 3(28) = 273. **E.**

12. $y=\sqrt{x^2-8x+16}=\sqrt{(x-4)^2}=\left|x-4\right|$, so the graph produced on the given interval is two right triangles. On the left of $x=4$ is a right triangle with base 5, height 4; on the right is a right triangle with base 4, height 4. The combined area is $(0.5)(5)(5)+(0.5)(4)(4)=20.5$, **B.**

13. The bounded area is a right triangle, with base (*x*-axis) 9 and height (*y*-axis) 3. Rotation around either axis will give a cone. Around the *x*-axis we have a radius of 9, so the volume is (1/3)(pi)(9)(9)=27pi. Around the *y*-axis we have a radius of 3, so the volume is (1/3)(pi)(81)(3)=81pi. The difference in volumes is 54pi, **A.**

14. Divide *x* and *y* by the coefficients on the other side, then add and square:

$$\begin{cases} x=2\cos t \\ y=3\sin t \end{cases} \to \begin{cases} \frac{x}{2}=\cos t \\ \frac{y}{3}=\sin t \end{cases} \to \frac{x^2}{4}+\frac{y^2}{9}=\cos^2 t+\sin^2 t=1.$$

The distance from the center to a focus, *c*, can be found by $c=\sqrt{a^2-b^2}$ . Here, $a=9, b=4$ , so $c=\sqrt{5}$. The distance focus to focus is therefore $2\sqrt{5}$, **A.**

15. For a logistic function $y=\frac{a}{1+be^{-x}}$, the point of inflection is located at $\left(\ln b, \frac{a}{2}\right)$.

We must put the given equation into this form, so we have $y=\frac{4}{1+\frac{3}{2}e^{-x}}$. Using the formula, we get $\left(\ln\frac{3}{2}, 2\right)$, **B.**

Also, the limiting value of $y=\frac{a}{1+be^{-x}}$ is $y=a$. The point of inflection is located halfway between $y=0$ and $y=a$, $y=\frac{a}{2}$. So, in order for $\frac{a}{1+be^{-x}}=\frac{a}{2}$,

$2=1+be^{-x}\to 1=be^{-x}\to b^{-1}=e^{-x}\to b=e^{x}\to \ln b=x$. To find *y*,

$$y=\frac{a}{1+be^{-\ln b}}=\frac{a}{1+\frac{b}{e^{\ln b}}}=\frac{a}{1+1}=\frac{a}{2}.$$

16. Statement I is false. Plugging in –2 into the first expression gives 1, and plugging into the second expression gives –1. Therefore, there is a jump there. Plugging in 1 into the second and third expressions gives 2, so II is true. Plugging 7 into the fourth expression gives 6, and the last expression is always 6; however, neither function is defined at $x=7$, so III is false. No work needs to be done for IV; all values around $x=9$ give a value of 6. Therefore, two of the statements are true, **B**.

17. $f(x)=\frac{1}{|2-x|+4}\rightarrow g(x)=f(-2x)=\frac{1}{|2+2x|+4}=\frac{1}{2|x+1|+4}$. $y=2|x+1|+4$ has its minimum at $(-1,4)$ so it has its maximum at $\left(-1,\frac{1}{4}\right)$. $g(x)$ has a horizontal asymptote at $y=0$ and it can be determined by cross-multiplication that the asymptote cannot be touched or crossed. The range is therefore $\left(0,\frac{1}{4}\right]$, **E.**

18. $\sin\left(3x+\frac{\pi}{12}\right)=\frac{1}{2}\rightarrow 3x+\frac{\pi}{12}=\frac{\pi}{6},\frac{5\pi}{6},\frac{13\pi}{6},\frac{17\pi}{6},\frac{25\pi}{6},\frac{29\pi}{6}\rightarrow x=\frac{\pi}{36},\frac{\pi}{4},\frac{25\pi}{36},\frac{11\pi}{12}$.
$\frac{1}{36}+\frac{25}{36}=\frac{13}{18}$, **A.**

19. Since the periods are the same, the largest possible value can be found by $\sqrt{\left(3\sqrt{5}\right)^2+\left(6\sqrt{7}\right)^2}=\sqrt{45+252}=\sqrt{297}=3\sqrt{33}$, **D.**

20. Using, $ax^2+bx+c=0$, we know that the sum of the roots can be found by $-b/a$ and that the product of the roots can be found by $c/a$. It is obvious from the graph that both roots are positive, so both their sum and their product is positive. We know that *a* is positive because the parabola opens up. That means that *b* must be negative and *c* must be positive. The only expressions that are guaranteed to be positive are *ac* and $a-b+c$, two expressions. **A**

21. $\lim_{x\to\infty}\frac{8+3^x}{4-3^x}=\lim_{x\to\infty}\frac{\frac{8}{3^x}+1}{\frac{4}{3^x}-1}=\lim_{x\to\infty}\frac{\frac{8}{3^\infty}+1}{\frac{4}{3^\infty}-1}=\frac{0+1}{0-1}=-1$ and

$\lim_{x\to-\infty}\frac{8+3^x}{4-3^x}=\lim_{x\to-\infty}\frac{\frac{8}{3^x}+1}{\frac{4}{3^x}-1}=\lim_{x\to-\infty}\frac{\frac{8}{3^{-\infty}}+1}{\frac{4}{3^{-\infty}}-1}=\frac{8\square3^\infty+1}{4\square3^\infty-1}=\frac{8}{4}=2$. $|-1+2|=1$, **D.**

22. The angle of inclination for $L_1$ is $90°$, and call the angle of inclination for $L_2$ $\theta$. Since we want the angle going from $L_1$ to $L_2$, the angle will have measure $\theta+90°$.
$\tan(\theta+90°)=-\cot\theta=-\frac{1}{\frac{1}{2}}=-2$. Since we want a positive angle, we must add $180°$ to the inverse expression $\tan^{-1}(-2)$, **C.**

23. Call the two "half angles" $\alpha_1$ and $\alpha_2$, with $L_1$ serving as one of the rays that creates $\alpha_1$. If the angle bisector has slope *m* and angle of inclination $\theta$, then the angle between the positive *y*-axis and the angle bisector, $\alpha_2$, will be $90°-\theta$. $\tan\alpha_2=\cot\theta=\frac{1}{\tan\theta}=\frac{1}{m}$.
$\tan\alpha_1=\frac{m-m_1}{1=mm_1}=\frac{m-2}{1+2m}$. Since both "half angles" are equal, $\frac{1}{m}=\frac{m-2}{1+2m}$. Solving the proportion we get $m^2-4m+1=0\rightarrow m=2\pm\sqrt{5}$, **A.**

24. A set of symmetric equations for a line in 3-space is in the form $\frac{x-x_0}{a}=\frac{y-y_0}{b}=\frac{z-z_0}{c}$, where *a*, *b*, and *c* are direction numbers. The direction numbers here can be 6, 4.5, –1.5; 12, 9, –3; or 4, 3, –1 (or any multiples). The coordinates of either point can be used for $x_0, y_0, z_0$. All four work, **D.**

25. $2(x-3)-1(y-0)-3(z-1)=0\rightarrow 2x-y-3z-3=0$, **D.**

26. $\overrightarrow{AB}=(6,0,-6), \overrightarrow{AC}=(-2,-2,-8), \overrightarrow{BC}=(-8,-2,-2)$, each having magnitude $\sqrt{72}$.
Using the equilateral triangle area formula $A=\frac{s^2\sqrt{3}}{4}$, we get $\frac{72\sqrt{3}}{4}=18\sqrt{3}$, **D.**

27. The volume is found by the triple scalar product, $V=\vec{a}\cdot\left(\vec{b}\times\vec{c}\right)$.

This product can be found by $\begin{vmatrix} 2 & 3 & 0 \\ 1 & 1 & 2 \\ 4 & 0 & -1 \end{vmatrix}=-2+24+0-0+3-0=25$, **B.**

28. $\frac{x^2}{9}+\frac{y^2}{4}=1\rightarrow 4x^2+9y^2=36;\ x=x'\cos\phi+y'\sin\phi,\ y=x'\sin\phi+y'\cos\phi$
$x=\frac{x'-y'}{\sqrt{2}},\ y=\frac{x'+y'}{\sqrt{2}}\rightarrow 4\left(\frac{(x-y)^2}{2}\right)+9\left(\frac{(x+y)^2}{2}\right)=36\rightarrow 4(x-y)^2+9(x+y)^2=72$
$4+9=13$, **A.**

29. Substitute $x=a$ and rewrite the expression in terms of powers of *m*:

$y=\left(m^2+4\right)a^2+(m-2)^2a-4m+2 \rightarrow a^2\left(m^2+4\right)+\left(m^2-4m+4\right)a-4m+2$

$y=m^2\left(a^2+a\right)-4m(a+1)+\left(4a^2+4a+2\right)$. Since the *y*-value, *b*, is a constant, then $a^2+a=0$ and $a+1=0$. This only occurs when $a=-1$. The corresponding *b*-value is 2. $a^2+b^2=(-1)^2+2^2=5$, **E.**

30. The center of the circle, $(h,k)$ must be in Quadrant III, so $h=k=-r$. Plugging these into the equation of a circle, we get $(-8-h)^2+(-1-k)^2=r^2 \rightarrow (-8+r)^2+(-1+r)^2=r^2$. This simplifies to $r^2-18r+65=0 \rightarrow (r-5)(r-13)=0 \rightarrow r=5,13$. The sum is 18, **C.**