1. Volume is given by $4/3\pi r^3$, surface area by $4\pi r^2$, where r is the radius. The ratio of these is r/3, which is (C) 4 when r = 12.

2. Extend FA and CB to meet at G. Extend BC and ED to meet at H. Extend AF and DE to meet at I. Then because the hexagon is equiangular, GHI is an equilateral triangle. Then GI = FE + AF + AB = 9. The area of an equilateral triangle with side length s is $s^2\sqrt{3} / 4$, so the area of GHI is $81\sqrt{3}/4$. Then we subtract the areas of ABG, CHD, and FEI, which are equilateral triangles of side length 3, 2, and 2 respectively. The area of the hexagon is then $81\sqrt{3}/4 - 9\sqrt{3}/4 - \sqrt{3} - \sqrt{3} =$ (B) $16\sqrt{3}$.

3. The tennis balls are stacked vertically, so the height of the cylinder is 3(2r) = 18. The volume of a cylinder with radius r and height h is $\pi r^2 h$, so when r=3 and h=18, the volume is 162π. The volume of the three spheres is $3(4/3\pi r^3) = 108\pi$, so the volume in question is $162\pi - 108\pi =$ (B) 54π.

4. Note that because AC and BC are tangents, OAC = OBC = 90 degrees. Then [ACBO] = [AOC] + [OBC]. Drawing OC, we see [AOC] = [OBC] because AO = OB = 6, AC = BC (tangents have equal length), and they share OC. Note that each triangle is 30-60-90, so if AO = 6, AC = $6\sqrt{3}$, so [AOC] = $\frac{1}{2}(6)(6\sqrt{3}) = 18\sqrt{3}$, so [ACBO] = 2[AOC] = (D) $36\sqrt{3}$.

5. Draw a rectangle with vertices (2,2), (-3,2), (-3,-2), and (2,-2). The area of this rectangle is 5x4 = 20. The triangles not included in the given pentagon have vertices {(-2,2) (-3,0) (-3,2)}, {(-3,0) (-1, -2) (-3, -2)}, and {(-1, -2), (2, -1), (2, -2)}. These have area 1, 2, and 3/2, respectively. So the total area is 20-1-2-3/2 = (B)15.5.

6. The x-intercept is at (2,0), and the y-intercept is at (0,4). The figure is a cone of radius 2 and height 4, which has volume $1/3\pi r^2 h =$ (E) $16\pi/3$.

7. Note that this triangle is right (if you divide the sides by 2, you get 7-24-25, a well-known right triangle with integer side lengths). The right angle subtends an arc of 180 degrees, so the hypotenuse goes through the center of the circle. So the radius of the circle is 50/2 = 25, so the area of the circle is (C)625π.

8. The graphs intersect when $|x|-b = x/2$, giving $x-b=x/2$ and $-x-b=x/2$, or $x=2b$ and $x=-2b/3$, or at the points (2b,b) and (-2b/3,-b/3). Note that the triangle is right ($y = |x|-b$ makes a right angle). The distance from this vertex (0, -b) to these two intersection points are $2\sqrt{2}b$ and $2\sqrt{2}/3b$, so the area is $\frac{1}{2}(2\sqrt{2}b)(2\sqrt{2}/3b) = 4b^2/3 = 12$, so b = (B) 3.

9. Draw the two radii that meet at the intersections of the chord with the circle. This forms an isosceles triangle with lengths 8, 8, and $8\sqrt{3}$. Drawing the altitude from the center of the circle to the chord, this divides the triangle into two 30-60-90 triangles (because one leg has length $4\sqrt{3}$ and the hypotenuse has length 8). So the two radii drawn cut off 120 degrees of the circle, which

is 1/3 the area of the circle, or $64\pi/3$. The triangle itself has area $1/2*8\sqrt{3}*4 = 16\sqrt{3}$, since the altitude has length 4. So the area in question is $64\pi/3 - 16\sqrt{3}$, or (B).

10. Consider an arbitrary circle. Inscribe an equilateral triangle inside. The formula for the circumradius is $R = abc/(4A)$, where a,b,c are side lengths for the triangle, R is the circumradius, and A is the area of the triangle. Now inscribe a circle inside that triangle. The formula for inradius is $r = A/s$, where A = area of the triangle, s = semiperimeter of the triangle. For an equilateral triangle, a=b=c, and $s = 3a/2$, so $r/R = 8A^2/(3a^4)$. In this case $A = a^2\sqrt{3}/4$, so $r/R = ½$, so the two circles' areas is ¼. Now given the equilateral triangle with side length 1, the inradius is $A/s = (\sqrt{3}/4)/(3/2) = \sqrt{3}/6$, so the area is $\pi/12$. The area of all the circles is an infinite series with first term $\pi/12$ and ratio ¼, which has sum $(\pi/12)/(1-1/4)$ = (D) $\pi/9$.

11. By the angle bisector theorem, AD/DC = AB/BC = 13/14. Since the triangles have the same height, the ratio of their areas is the ratio of the lengths of their bases, which is DC/AD = (A) 14/13.

12. WLOG the two points are the base of the triangle. If we want a certain area, then it must be determined by the height. So the variable point must lie on a line parallel to the line determined by the two fixed points, and at a certain distance. There are an infinite number of lines that are some distance from this fixed line, determined namely by (E) an infinite length cylinder.

13. It is easiest to note that the area of a quadrilateral determined by connecting the midpoints of a quadrilateral is half the area of the original quadrilateral. There are a couple of ways to find the area of the parallelogram, probably the easiest of which is $[ABCD] = 4*5*\sin(60) = 10\sqrt{3}$, so the area in question is (B) $5\sqrt{3}$.

14. The volume of water always retains a cone shape similar to the cone itself. Thus at volume 12π, we can write the radius as 6/k, and the height as 8/k for some k. The volume of the water is $1/3\pi(6/k)^2(8/k) = 96\pi/k^3$. This is 12π at k = 2, or r = 3 and h = 4. If the water rises one unit, then h = 5, and r = (5/4)*3 = 15/4. The new volume is $1/3\pi(15/4)^2(5)$ = (B) $375\pi/16$.

15. Note that the side lengths of successive rectangles is determined by the Fibonacci sequence, $F_1 = 1$, $F_2=1$, and $F_n=F_{n-1}+F_{n-2}$. If there are 11 squares, this corresponds to a $F_{11}xF_{12}$ rectangle, or 89x144 = (D) 12816.

16. By the triangle inequality, the range of possible lengths for the longest side is 5 to 6. Then the possible side lengths are {5,5,4} {6,6,2}, {6,5,3}, {6,4,4}. The area of these can be found by Heron's formula (or some other method): AREA = $\sqrt{(s(s-a)(s-b)(s-c))}$, where s is the semi-perimeter and a,b,c are the side lengths. The smallest triangle is {6,6,2}, which has area (E) $\sqrt{35}$.

17. Note that BAF is similar to BOD. Also note that since ABC is isosceles, AF = $2\sqrt{3}$, so BF = $2\sqrt{6}$. If we call r the radius of the circle, then by similar triangles, $6/(2\sqrt{6} + r) = 2\sqrt{3}/r$, which yields $r = 3\sqrt{2} + \sqrt{6}$. Then BO = $2\sqrt{6} + r = 3\sqrt{2} + 3\sqrt{6}$. By the Pythagorean Theorem, this yields BD =

6+2√3. Then the area of the quadrilateral is twice the area of BDO, or 2(1/2*(6+2√3)*(3√2 + √6)) = (C) 24√2 + 12√6.

18. The matrix that preserves area must have determinant 1. The determinant of a matrix [a b][c d] is ad-bc. By inspection, only (A) has determinant 1 (7*3 – 4*5).

19. The distance between (8,3) and (64/5, -17/5) is 8 by the distance formula. The distance between (2,11) and (64/5, -17/5) is 18 by the distance formula. Given any point on the ellipse, the distance between that point and (8,3) plus the distance between that point and (2,11) is 18+8 = 26 by definition of an ellipse. Now consider the end of the minor axis of the ellipse. This point, the center of the ellipse, and the point (2,11) form a right triangle with leg 5 and hypotenuse 26/2 = 13 (the leg is 5 because the center of the ellipse is the midpoint of the two foci, and the distance between the foci is 10). Then by the Pythagorean Theorem, the minor axis has length 12. Now consider the end of the major axis of the ellipse, the point closer to (2,11) than (8,3). If the distance between this point and (2,11) is k, then k+(10+k) = 26, since this point is on the ellipse (10+k is the distance from (8,3) to this point). Then k=8, and the major axis has length 8+5 = 13. The area of the ellipse is then π*12*13 = (B) 156π.

20. An icosahedron has 20 equilateral triangles as a surface. So the surface area is $20(8^2\sqrt{3}/4)$ = (B) 320√3.

21. OAC and OBC are right and congruent. Therefore AC = BC = 8 by the Pythagorean Theorem. Then the area of the quadrilateral is [OAC]+[OBC] = 2(1/2*6*8) = (C) 48.

22. Note that any tetrahedron satisfying these properties is the same. To see this, since we have 6 faces and 4 points to choose from, by the Pigeonhole Principle, two points must be on opposite faces. Choosing a third point always yields the same resulting triangle, and the fourth point cannot be opposite the third point (otherwise all four points are coplanar). This leaves two possible points, and both yield identical tetrahedron. Now orient the tetrahedron so that there is a right triangle in the middle layer and a point on the top face. The right triangle is isosceles and has legs of length 4√2, yielding a base of area 16. The height is half the length of the cube, or 4. The volume of a pyramid is 1/3bh, b=area of base, h = height, so the volume of the tetrahedron is 1/3*(16)*(4) = (E) 64/3.

23. Angle A is trisected into 15 degree angles. The ratio of their areas is the same as the ratio of BD:EC, since the triangles' heights are the same. Doing a bit of angle chasing, ADB = 75 degrees, ADE = 105 degrees, AED = 60 degrees, AEC = 120 degrees. By the law of sines on triangle AEC, EC/sin(15) = AC/sin(120) = AB√2/sin(120), so EC = AB√2sin(15)/sin(120). By the law of sines on triangle ABD, BD/sin(15) = AB/sin(75), so BD = ABsin(15)/sin(75). Then BD/EC = sin(120)/(√2sin(75)). sin(75) is (√6 + √2)/4, so the answer is (D) (3 - √3)/2.

24. Draw a triangle with two vertices as the centers of two of the large circles, and the last vertex the center of the small circle. This triangle has side lengths (1+r), (1+r), and 2, where r is the radius

of the small circle. The angle between the two equal sides is 120 by symmetry of the large circles. Drawing the altitude from the center of the small circle to the side of length 2, we get two 30-60-90 triangles. The side opposite the 60 degree angle has length 1, so the hypotenuse (or the side with length 1+r) has length $2\sqrt{3}/3 = 1+r$. Then $r = (2\sqrt{3} - 3)/3$, so the area is $\pi r^2 = ((7-4\sqrt{3})/3)\pi$, or (C).

25. WLOG let A be the right angle. Suppose D is on the same side of BC as A. Then the maximum area occurs when D is halfway between A and B or A and C. If D is on this point, consider the equilateral triangle ABD. If we draw the altitude from D to AB, we get two right triangles, with a side length of 3 opposite a 67.5 degree angle. By half-angle formulas, we can get $\sin(67.5) = \sqrt{(2+\sqrt{2})}/2$. By law of sines, $h/\sin(90) = 3/\sin(67.5) = 3\sqrt{(4-2\sqrt{2})}$. The area of ADB is then $½*h*h*\sin(135) = 9(\sqrt{2} - 1)$. The total area of the quadrilateral is then $18 + 9(\sqrt{2} - 1) = 9\sqrt{2} + 9$, which is less than $9*1.5 + 9 = 22.5$. So D can lie anywhere on this side of the circle.

As D moves farther from BC, the area gets larger. So we want to find the point at which the area is 27 exactly. This requires finding the point D when $[BDC] = 27 - 18 = 9$. Note that BDC is 90 degrees, since BC is the hypotenuse. We know $BC = 6\sqrt{2}$, so we want $½*6\sqrt{2}*h = 9$, or $h = 3\sqrt{2}/2$. So let the altitude have length $3\sqrt{2}/2$, and let the altitude intersect BC at E. Note that BDE ~ DCE. Call EC = x, so $BE = 6\sqrt{2} - x$. Then by similar triangles, $(3\sqrt{2}/2)/x = (6\sqrt{2} - x)/(3\sqrt{2}/2)$, which yields the equation $2x^2-12\sqrt{2}x+9=0$ after some simplification. By the quadratic formula this yields the solutions $x = 3\sqrt{2} + 3\sqrt{6}/2$ and $x = 3\sqrt{2} - 3\sqrt{6}/2$. Either one works because of symmetry on the arc BC. Take $x = 3\sqrt{2} - 3\sqrt{6}/2$. Call DCE angle b. Then $\tan(b) = (3\sqrt{2}/2)/(3\sqrt{2} - 3\sqrt{6}/2) = 2+\sqrt{3}$. Then b = 75 degrees, so BOD is 150 degrees, where O is the center of the circle. Then DOC is 30 degrees. By symmetry, there is another 30 degree region closer to B in which the quadrilateral will have area less than 27 if D is placed there. So there is a total of 180+30+30 = 240 degrees, so the probability is 240/360 = (A) 2/3.

26. Let the octagon have vertices ABCDEFGH. If two vertices are next to each other, say A and B are chosen, then ABC, ABD, and ABE have distinct area. If the minimum distance between two vertices is increased by a vertex, say A and C are chosen, then ACE and ACF have distinct area. All other areas are congruent to one of these, so the answer is (C) 5.

27. Suppose the prism has side lengths a,b,c. We are given $\sqrt{(a^2+b^2+c^2)} = 60$, or $a^2+b^2+c^2=3600$. We are also given $4(a+b+c) = 336$, or $a+b+c = 84$. Then $(a+b+c)^2 = 7056$. Expanding, $a^2+b^2+c^2+2(ab+bc+ca) = 7056$, so $2(ab+bc+ca) =$ (E) 3456, which is the surface area.

28. Consider the triangle of the notecard not covered by the folded over region. The long leg has length 3, call the short length h. Then the hypotenuse has length 5-h because the leg and hypotenuse share the length of the notecard. So $5-h = \sqrt{(9+h^2)}$ which yields $h=8/5$. The area of this triangle is then $½*3*8/5 = 12/5$.

Now the folded over region is a quadrilateral. One side has two vertices that are vertices of the original note card, call them A and B, where A is the vertex that met the opposite corner of the notecard. Labeling the other vertices in order, call them C and D. Draw a line parallel to AB from C meeting AD at E. Now the quadrilateral is divided into a rectangle ABCE and a right triangle DEC. Note that by symmetry, BC = 8/5 as well, so [ABCE] = 8/5*3 = 24/5. Also DE = 5-8/5-AE = 5-8/5-8/5 = 9/5, so [DEC] = 1/2*(9/5)*3 = 27/10. So the total area is 12/5 + 24/5 + 27/10 = (A) 99/10.

29. Note that the roots are actually an octagon inscribed inside the unit circle. (the roots of $x^n = -1$ form an n-gon inscribed inside the unit circle.) This octagon has area $2\sqrt{2}$.

30. Each hour is 360/12 = 30 degrees of the clock. The minute hand is ¾(360) = 90 degrees counter-clockwise from the 12 o'clock mark. The hour hand is 2(30) = 60 degrees clockwise from the 12 o'clock mark, plus ¾(30) = 22.5 more degrees clockwise because ¾ of the hour has passed. The angle subtended is 22.5+60+90 = 172.5 degrees. This is 172.5/360 = (C)23/48 of the area of the clock.

TIEBREAKER: Archimedes