Note: For all questions, answer "(E) NOTA" means none of the above answers is correct.

1. Solve for x: $3^{2x+2} = 5^{6x+3}$

   (A) $\frac{5\ln 3 - 2\ln 3}{6\ln 5 - 2\ln 3}$ (B) $\frac{3\ln 5 - 2\ln 6}{2\ln 3 - 6\ln 5}$ (C) $\frac{3\ln 5 - 2\ln 3}{2\ln 3 - 5\ln 6}$ (D) $\frac{3\ln 5 - 3\ln 2}{2\ln 3 - 6\ln 5}$ (E) NOTA

2. Consider the following matrices: $A = \begin{bmatrix} \log x & 1 \\ 0 & 2 \end{bmatrix}$, $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$, $L = \begin{bmatrix} 2\log x & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}$, and S is the 2×2 Identity Matrix. Determine the product of all values of x such that the matrix product $ALISSA$ has no inverse.

   (A) $e^{-1/2}$ (B) $e^{-1/4}$ (C) $e^{-1/8}$ (D) $e^{-1/16}$ (E) NOTA

3. The *Hyperbolic Sine Function*, denoted sinh, is defined as $\sinh x = \frac{1}{2}(e^x - e^{-x})$. Find $\sinh^{-1} 5$; in other words, the inverse of the Hyperbolic Sine Function evaluated at 5.

   (A) $\ln\left(\sqrt{26} - 1\right)$ (B) $\ln\left(1 + \sqrt{26}\right)$ (C) $\ln\left(\sqrt{26} - 5\right)$ (D) $\ln\left(\sqrt{26} + 5\right)$ (E) NOTA

4. Given that x, y, and z are all greater than 1, simplify $3\log_4 x - 4\log_2 y + \log_8 z$ into a single logarithm.

   (A) $\log_{64}\left(\frac{x^9z^2}{y^{24}}\right)$ (B) $\log_{64}\left(\frac{x^3z}{y^4}\right)$ (C) $\log_{32}\left(\frac{x^6z^2}{y^{12}}\right)$ (D) $\log_{32}\left(\frac{x^3z}{y^{24}}\right)$ (E) NOTA

5. If $x^2 + y^2 = 14xy$ for positive x and y, then $\log\left(k(x+y)\right) = \frac{1}{2}(\log x + \log y)$ for some constant k. Find the value of k.

   (A) 1/8 (B) 1/4 (C) 1/2 (D) 1 (E) NOTA

6. Find the value(s) of α such that $\ln(\sin(\alpha + \pi)) - \ln(\cos(\alpha + \pi)) = 0$, where $\alpha \in \left(0, \frac{3\pi}{2}\right)$.

   (A) $\frac{\pi}{4}, \frac{5\pi}{4}$ (B) $\frac{3\pi}{4}$ (C) $\frac{5\pi}{4}$ (D) $\frac{3\pi}{4}, \frac{7\pi}{4}$ (E) NOTA

7. The graph of $y = 4(e^{-x} + 2)^{-1}$ has horizontal asymptotes at $y = J$ and $y = K$. Find the value of $J + K$.

   (A) 0 (B) 2 (C) 4 (D) 6 (E) NOTA

8. Find the number of digits in the base-10 expansion of 2013^{1000}.

(A) 3303 (B) 3304 (C) 3305 (D) 3306 (E) NOTA

9. A radioactive particle has a half-life of 24 hours, and after 1024 days has reduced in mass to $2^{(-2^8)}$ grams. What was the particle's starting mass? Express your answer in grams.

(A) 2^{256} (B) 2^{512} (C) 2^{768} (D) 2^{896} (E) NOTA

10. Define a recursive sequence such that $a_1 = \sqrt{2}$, and for $n \geq 1$, $a_{n+1} = \left(\sqrt{2}\right)^{a_n}$. Which of the following is closest to the value of $a_{10{,}000{,}000{,}000{,}000{,}000{,}000{,}000{,}000{,}000{,}000{,}000}$?

(A) 2 (B) 4 (C) 8 (D) 16 (E) NOTA

11. Find the sum of the fourth powers of all real numbers x satisfying $x^{x^2-5x+6} = 1$.

(A) 18 (B) 34 (C) 99 (D) 274 (E) NOTA

12. For real number u, let $0 < \arccos u < \pi$. Solve for x: $e^{\tan \arccos\left(-\frac{3}{5}\right)} = x^e$.

(A) $e^{-\frac{3}{4e}}$ (B) $e^{-\frac{4}{5e}}$ (C) $e^{-\frac{4}{3e}}$ (D) $e^{-\frac{5}{3e}}$ (E) NOTA

13. Let $i = \sqrt{-1}$. Given that $e^{ix} = \cos x + i \sin x$, which of the following equals $\tan x$?

(A) $\frac{-i(e^{ix}-e^{-ix})}{e^{ix}+e^{-ix}}$ (B) $\frac{i(e^{ix}-e^{-ix})}{e^{ix}+e^{-ix}}$

(C) $\frac{-i(e^{ix}-e^{-ix})}{2(e^{ix}+e^{-ix})}$ (D) $\frac{2(e^{ix}+e^{-ix})}{i(e^{ix}-e^{-ix})}$ (E) NOTA

14. For positive integer n, define a function f such that $f(n) = \log_9 n$ if $\log_9 n$ is rational and $f(n) = 0.10$ otherwise. Evaluate: $\sum_{n=1}^{2013} f(n)$

(A) 10.50 (B) 211.10 (C) 605.50 (D) 1092.00 (E) NOTA

15. Solve for x: $\frac{3e^{3x}-e^{2x}-e^x-4}{e^x+1} = 0$

(A) 0 (B) $\ln 2$ (C) $\ln 3$ (D) $\ln 4$ (E) NOTA

16. Find the value of x if $4^{\frac{x}{y}+\frac{y}{x}} = 32$ and $\log_3(x+y) + \log_3(x-y) = 1$.

(A) 3 (B) 4 (C) 5 (D) 8 (E) NOTA

17. Let $i = \sqrt{-1}$. Which of the following is equal to $2\ln i$?

(A) 0 (B) $i\pi$ (C) $2i\pi$ (D) $3i\pi$ (E) NOTA

18. Let N equal the number of zeroes at the end of 2013! when expressed in base 10. Find the value of $\log_{167}\left(\frac{x}{3}\right)$.

(A) $\log_{167} 134$ (B) 1 (C) $\log_{167}\frac{482}{3}$ (D) $\log_{167}\frac{498}{3}$ (E) NOTA

19. Find the value of xy if $\log_8 x + \log_4 y^2 = 5$ and $\log_8 y + \log_4 x^2 = 7$.

(A) 1024 (B) 512 (C) 256 (D) 128 (E) NOTA

20. Given that $S = \sum_{n=1}^{2012}(\ln(\sum_{k=1}^{n} k))$, evaluate: e^S

(A) $\frac{(2012!)(2013!)}{2^{2012}}$ (B) $\frac{(2012!)(2013!)-1}{2^{2011}}$

(C) $\frac{(2012!)(2013!)-2}{2^{2013}}$ (D) $\frac{(2013!)(2012!)-3}{2^{2014}}$ (E) NOTA

21. Given that x and y are nonnegative reals, find the sum of x and y, given that $3x + y = 6$ and that e^{xy} is as large as possible.

(A) 1 (B) 2 (C) 3 (D) 4 (E) NOTA

22. Find the value of q given that $(\log_3 p)^2 = \log_3 p^2$ and $\log_3(p+q) = \log_3 p + \log_3 q$.

(A) 9/8 (B) 10/9 (C) 11/10 (D) 12/11 (E) NOTA

23. Find the vertical asymptotes of the graph of $y = \log\left(\frac{x^2-3x-10}{x+2}\right)$.

(A) $x = 2$ (B) $x = 4$ (C) $x = 5$ (D) $x = -2$ (E) NOTA

24. Find the equation of the horizontal asymptote(s) of the graph of $y = e^{\arctan x}$.

(A) $y = \pm 1$ (B) $y = \pm e^{\frac{\pi}{4}}$ (C) $y = e^{\pm\frac{\pi}{2}}$ (D) $y = 0$ (E) NOTA

25. Given that x is a real number, find the domain of $f(x) = \log\left(\frac{|x+1|}{x^2+3x+2}\right)$.

(A) $x \notin \{-2, -1\}$
(B) $x \in (-\infty, -2) \cup (-1, \infty)$
(C) $x \in (-2, -1)$
(D) $x \in (-\infty, \infty)$
(E) NOTA

26. Let x, y, and z be positive real numbers such that $x^{\log_2 7} = 8$, $y^{\log_3 5} = 81$, and $z^{\log_5 216} = \sqrt[3]{5}$. Find the value of $x^{(\log_2 7)^2} + y^{(\log_3 5)^2} + z^{(\log_5 216)^2}$.

(A) 8836
(B) 6630
(C) 3240
(D) 974
(E) NOTA

27. Find the coefficient of xyz^4 in the expansion of $(x + 3y + 2z)^6$ with like-terms combined.

(A) 160
(B) 480
(C) 1440
(D) 4320
(E) NOTA

28. If $\log 49 = a$ and $\log 625 = b$, express $\log\frac{1}{28}$ in terms of a and b.

(A) $\frac{a-b}{2} + 2$
(B) $\frac{b-a}{2} + 2$
(C) $\frac{a-b}{2} - 2$
(D) $\frac{b-a}{2} - 2$
(E) NOTA

29. If the solutions to the system of equations given by $\log_{4096} x + \log_{2013} y = 2$ and $\log_x 4096 - \log_y 2013 = 1$ are (x_1, y_1) and (x_2, y_2), find the value of $\log_4(x_1 y_1 x_2 y_2)$.

(A) 36
(B) 30
(C) 24
(D) 18
(E) NOTA

30. Evaluate: $\ln e + \ln\sqrt{e} + \ln\sqrt[4]{e} + \ln\sqrt[8]{e} + \cdots$

(A) 16
(B) 8
(C) 4
(D) 1
(E) NOTA