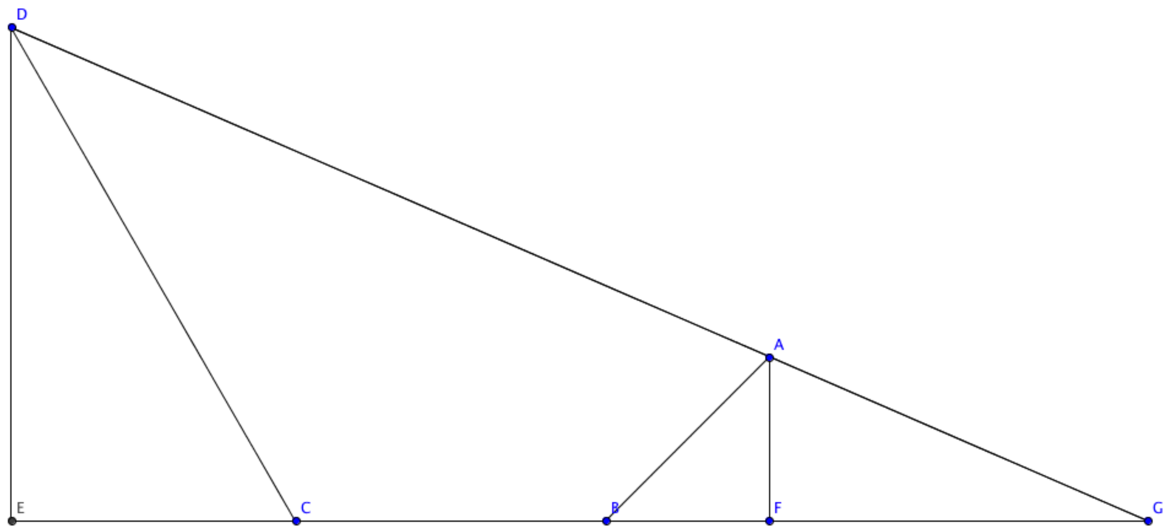
Question	Solution
P1.	The common difference of an arithmetic sequence is the difference between a term and the previous term (in that order); in this case, $23-(-10)=\mathbf{33}$.
P2.	Since $2^{15}=32768$, the digital sum is $3+2+7+6+8=\mathbf{26}$.
P3.	The smaller value is $k=\mathbf{-4}$.
P4.	We have $\csc(330°)=\mathbf{-2}$.
P5.	We have $(A-B)^{D-C}=(33-26)^{-2-(-4)}=7^2=\mathbf{49}$.
1.	If $x=3$, then $\lvert 3-2\rvert<\lvert 3-6\rvert$, or $1<3$, which is true. However, if $x=4$, then $\lvert 4-2\rvert<\lvert 4-6\rvert$, or $2<2$, which is false. The largest integer solution is therefore $x=\mathbf{3}$.
2.	The period of $y=\sin\left(\frac{\pi x}{6}\right)$ is $\frac{2\pi}{\pi/6}=12$. The absolute value cuts the period in half, since portions below the x-axis get reflected to positive values, so the graph starts the cycle quicker. The answer is **6**.
3.	We have $8\sqrt{2}\cos\frac{7\pi}{4}-6\sqrt{3}\sin\frac{4\pi}{3}=8\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)-6\sqrt{3}\left(\frac{-\sqrt{3}}{2}\right)=8+9=\mathbf{17}$.
4.	We have $4^x-4^{x-1}=4^x-\left(\frac{1}{4}\right)4^x=\left(\frac{3}{4}\right)4^x=24$, so $4^x=2^{2x}=32=2^5$, so $2x=5$. Therefore, $(2x)^{2x}=5^5=\mathbf{3125}$.
5.	We have $AB-C+D=(3)(6)-17+3125=\mathbf{3126}$.
6.	Since $2^{12}<5566<2^{13}$, the binary representation of 5566 is a 1 followed by 12 other digits for a total of **13** digits.

7.	For convenience's sake, let square $ABCD$ have a side length of 2. We know that triangles AED and DFC are congruent. Let $\phi = \angle EDA = \angle FDC$ so that $\theta = \frac{\pi}{2} - 2\phi$; thus, $\sin\theta = \sin\left(\frac{\pi}{2} - 2\phi\right) = \cos(2\phi) = 1 - 2\sin^2\phi$. Since $\sin\phi = 1/\sqrt{5}$, we have $\sin\theta = 1 - 2\sin^2\phi = 1 - 2\left(\frac{1}{\sqrt{5}}\right)^2 = 1 - \frac{2}{5} = \mathbf{\frac{3}{5}}$.
8.	If $\cos x = \frac{\sqrt{3}}{5}$, then $\cos^2 x = \frac{3}{25}$ and $\sin^2 x = 1 - \frac{3}{25} = \frac{22}{25}$. Consequently, $\cot^2 x = \frac{3/25}{22/25} = \frac{3}{22}$, making the answer $484\left(\frac{3}{22}\right) = \mathbf{66}$.
9.	The relation can be expressed as $f = Kg^2h$ for some K, which, in this case, is equal to $K = \frac{f}{g^2h} = \frac{128}{4^2\times 2} = 4$. The answer is then $f = 4(3^2)(6) = \mathbf{216}$.
10.	We have $AB(D - C) = (13)\left(\frac{3}{5}\right)(216 - 66) = \mathbf{1170}$.
11.	If $L(x) = mx + b$, then $I(x) = \frac{x-b}{m}$. So we have $mx + b = \frac{4(x-b)}{m} + 3$, or $mx + b = \frac{4x}{m} + 3 - \frac{4b}{m}$. Set corresponding coefficients equal to each other to obtain the equations $m = 4/m$ and $b = 3 - \frac{4b}{m}$. Since the slope is positive, $m = 2$. Plug this into the second equation to get $b = 3 - \frac{4b}{2} = 3 - 2b$, or $b = 1$. Thus, $L(10) = 2(10) + 1 = \mathbf{21}$.
12.	By the Power-Reducing formula $\cos^2 x = \frac{1+\cos(2x)}{2}$, we have $\cos^2 x = \frac{1+\frac{3}{7}}{2} = \frac{5}{7}$, so that $m + n = \mathbf{12}$.
13.	Since $\sin\frac{11\pi}{6} = -\frac{1}{2}$, we have $\arcsin(-\frac{1}{2}) = -\mathbf{\frac{\pi}{6}}$.
14.	For positive x, $g^{-1}(x) = x^2 - 2$ (this is easy to check just by plugging it into g and seeing that the identity function is obtained), so $g^{-1}(5) = 23$ and $f(g^{-1}(5)) =$

	$f(23) = \mathbf{65}$.
15.	We have $AB\cot^2 C + 10D = (21)(12)\cot^2\left(-\frac{\pi}{6}\right) + 10(65) = \mathbf{1406}$.
16.	By inspection, $x = \mathbf{1}$.
17.	We have $\sin 20° (\tan 10° + \cot 10°) = 2\sin 10° \cos 10° \left(\frac{\sin 10°}{\cos 10°} + \frac{\cos 10°}{\sin 10°}\right) =$ $2(\sin^2 10° + \cos^2 10°) = \mathbf{2}$.
18.	The polar graph is a circle with diameter 10. So the radius is 5 and the area is $\mathbf{25\pi}$.
19.	By inspection, $x = 1$ is a solution. Via Synthetic Division or favorite factoring method, $x^3 - 7x + 6 = (x-1)(x-2)(x+3)$, so $x = 2$ is the other positive root. The desired product is therefore equal to $\mathbf{2}$.
20.	We have $D\left(A + \sin\frac{C}{B}\right) = 2\left(1 + \sin\frac{25\pi}{2}\right) = 2(1+1) = \mathbf{4}$.
21.	Hexagonal numbers have a second-level common difference of $6 - 2 = 4$ and octagonal numbers have a second-level common difference of $8 - 2 = 6$. Thus, the hexagonal numbers are $1, 6, 15, 28, \ldots$ and the octagonal numbers are $1, 8, 21, 40, \ldots$. The answer is $15 + 40 = \mathbf{55}$.
22.	Let $x = \cos t$ and $y = \sin t$; this is a legal substitution since $x^2 + y^2 = 1$. Note that $x + y = \cos t + \sin t = \sqrt{2}\sin\left(t + \frac{\pi}{4}\right)$, so the maximum value of $x + y$ is $\sqrt{2}$. Thus, the maximum value of $2(x+y)^3$ is $2\left(2^{\frac{3}{2}}\right) = 2 \times 2\sqrt{2} = \mathbf{4\sqrt{2}}$.
23.	All arguments are in degrees. We know that valid angles are of the form $53 + 360n$ or $127 + 360n$ for integer n. Setting $53 + 360n > 775$ yields $n > 2.00556\ldots$ so pick $n = 3$, resulting in an angle of 1133 degrees. However, setting $127 + 360n > 775$

	yields $n > 1.8$, so pick $n = 2$, yielding an angle of $127 + 360(2) = \mathbf{847}$ degrees.
24.	If $\log_k 3 \leq 4$, then $k^4 \geq 3$, or $k^8 \geq 9$, making the answer **9**.
25.	We have $(B+C)\left(A - \sum_{j=1}^{D+1} j\right) = (B+C)\left(55 - \sum_{j=1}^{9+1} j\right) = (B+C)(55-55) = \mathbf{0}$.
26.	Notice that vectors **a**, **b**, and **c** are mutually orthogonal to each other. Let $\mathbf{d} = [-6, -17, 6]$. We have $c_1 = \frac{\mathbf{d}\cdot\mathbf{a}}{\mathbf{a}\cdot\mathbf{a}} = \frac{12}{2} = 6$, $c_2 = \frac{\mathbf{d}\cdot\mathbf{b}}{\mathbf{b}\cdot\mathbf{b}} = \frac{-68}{34} = -2$, and $c_3 = \frac{\mathbf{d}\cdot\mathbf{c}}{\mathbf{c}\cdot\mathbf{c}} = \frac{51}{17} = 3$, so $c_1 c_2 c_3 = \mathbf{-36}$.
27.	If n is an integer, we have the identities $\sin(2\pi x) = \cos\left(\frac{\pi}{2} - 2\pi x + 2\pi n\right) = \cos\left(\frac{3\pi}{2} + 2\pi x + 2\pi n\right) = \cos(3\pi x)$, leading to the equations $3\pi x = \frac{\pi}{2} - 2\pi x + 2\pi n$ and $3\pi x = \frac{3\pi}{2} + 2\pi x + 2\pi n$. The first equation has solutions in the interval of .10, .50, 1.90, 1.30, and 1.70. The second equation has a single solution in the interval of 1.5. The sum of all these x-values is **6**.
28.	Forget what's inside the sine function and focus on the outer coefficient. The amplitude is **2013**.
29.	The distance between the centers of the circles, $(3, 12)$ and $(-4, -12)$, is $\sqrt{7^2 + 24^2} = 25$. The sum of the radii of the circles is $12 + 13 = 25$. Thus, the circles intersect at exactly one point, which has area **0**.
30.	The determinant of $\begin{bmatrix} C & B \\ A & D \end{bmatrix}$ is $CD - AB = C(0) - (-36)(6) = \mathbf{216}$.
31.	The coordinates of triangle POQ are $(0,0)$, $(5,0)$, and (x,y), where $x^2 + y^2 = 36$. The centroid of POQ is the average of the coordinates, or $\left(\frac{x+5}{3}, \frac{y}{3}\right)$. Suppose $\left(\frac{x+5}{3}, \frac{y}{3}\right) = (a,b)$ so that $\frac{x+5}{3} = a$ and $\frac{y}{3} = b$. Solving each equation for x and y, squaring both sides, and adding the equations, we arrive at $(3a-5)^2 + (3b)^2 = 36$,

	or $\left(a-\frac{5}{3}\right)^2+b^2=4$, an equation of a circle of radius 2. The desired area is $\boldsymbol{4\pi}$.
32.	We want the angle opposite the side with length 6. Let this angle equal θ. By the Law of Cosines, $\cos\theta=\frac{7^2+8^2-6^2}{2\times7\times8}=\frac{11}{16}$. Thus, $m+n=11+16=\mathbf{27}$.
33.	We have $h(x)=f(x)g(x)=\sin x\cos x=\frac{1}{2}\sin(2x)$, so $h\left(\frac{\pi}{8}\right)=\frac{1}{2}\sin\frac{\pi}{4}=\frac{1}{2}\frac{\sqrt{2}}{2}=\boldsymbol{\frac{\sqrt{2}}{4}}$.
34.	Since $4096^2=4^{12}, 8^4=2^{12}, 81^3=3^{12}$, and $25^6=5^{12}$, the smallest element in D is 8^4, so $f(m)=f(8^4)=\mathbf{100}$.
35.	We have $\frac{AD}{\pi}(\sin 15°+C)^2+B=\frac{(4\pi)(100)}{\pi}\left(\frac{\sqrt{6}-\sqrt{2}}{4}+\frac{\sqrt{2}}{4}\right)^2+27=400\left(\frac{6}{16}\right)+27=$ $\mathbf{177}$.
36.	The equation is of the form $M=PDP^{-1}$, where D is a diagonal matrix. Therefore, we have $M^{10}=(PDP^{-1})^{10}=PD^{10}P^{-1}$. Notice that $D^{10}=\begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix}^{10}=$ $\begin{bmatrix}(-1)^{10} & 0\\ 0 & 1^{10}\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ is the identity matrix I. Thus, $M^{10}=PD^{10}P^{-1}=PIP^{-1}=$ $PP^{-1}=I$. The sum of the elements of the 2×2 identity matrix is $\mathbf{2}$.
37.	Note that $\frac{x}{100\pi}-1$ is less than -1 whenenver $x<0$ and greater than 1 when $x>200\pi$. Thus, the two graphs will have intersection points on the interval $x\in[0,200\pi]$, in which the graph of $y=\sin x$ will exhibit 100 full cycles. We subtract 1 from this total to account for the double-counting of intersection points in the middle of the interval. The answer is $\mathbf{199}$.
38.	Two consecutive angles of a parallelogram add up to π. Thus, $\cos\pi=\mathbf{-1}$.
39.	The sum of the digits of 10^1-1 is $9\times1=9$. The sum of the digits of $(10-1)(10^2-1)=891$ is $9\times2=18$. Basically, adding another term in the product in

	accordance with the pattern creates a bunch of new digits whose sum is 9, with the number of "bundles" of digits whose sum is 9 equal to the number of 9s in the largest factor. Specifically, the sum of the digits of $(10-1)(10^2-1)(10^4-1)(10^8-1)$ is $9 \times 8 = \mathbf{72}$ because 10^8-1 has eight 9s in its base-10 representation.
40.	We have $\frac{B-C}{A} + D = \frac{199--1}{2} + 72 = \mathbf{172}$.
41.	 Starting with quadrilateral $ABCD$, draw auxiliary line segments to obtain the diagram above, where AF and DE are perpendicular to EG, which contains the points C, B, and F. Triangle ABF is a 45-45-90 triangle, so $AF = BF = \frac{\sqrt{6}}{\sqrt{2}} = \sqrt{3}$. Triangle DCE is a 30-60-90 triangle, so $EC = 3$ and $DE = 3\sqrt{3}$. Triangles AFG and DGE are similar. Therefore, $\frac{AF}{DE} = \frac{FG}{EG}$, or $\frac{\sqrt{3}}{3\sqrt{3}} = \frac{FG}{3+5-\sqrt{3}+\sqrt{3}+FG} = \frac{FG}{8+FG}$, or $FG = 4$. Moreover, $AG = \sqrt{AF^2 + FG^2} = \sqrt{\sqrt{3}^2 + 4^2} = \sqrt{19}$. Based on the earlier equation, triangles AFG and DGE are in a 3-to-1 linear ratio, so $AD = 2 \times AG = \mathbf{2\sqrt{19}}$.
42.	Perhaps the fastest way to do this problem is to know in advance that $\cos\frac{\pi}{5} = \frac{1+\sqrt{5}}{4}$; hence the minimal polynomial will also have $\frac{1-\sqrt{5}}{4}$ as a root. The two roots have a

	sum of $\frac{1+\sqrt{5}+1-\sqrt{5}}{4}=\frac{1}{2}$ and product of $\left(\frac{1+\sqrt{5}}{4}\right)\left(\frac{1-\sqrt{5}}{4}\right)=\frac{1-5}{16}=\frac{-1}{4}$, leading to a minimal polynomial of $P(x)=x^2-\frac{1}{2}x-\frac{1}{4}$ or after making all the coefficients integers, $P(x)=4x^2-2x-1$. Hence, $P(-1)=4(-1)^2-2(-1)-1=4+2-1=\mathbf{5}$.
43.	The exterior angles of a polygon add up to 2π. Thus, $\sin(2\pi)=\mathbf{0}$.
44.	The entry in the second row, second column of M^{-1} is the second row, second column of the transposed adjoint matrix of M, divided by $\lvert M\rvert$. The second row, second column cofactor of M is $(-1)^{2+2}\begin{vmatrix}-1 & -5\\ 4 & 5\end{vmatrix}=15$. Thus, $\frac{15}{4x-47}=15$, so $x=\mathbf{12}$.
45.	We have $A^2+B^2+C^2-D^2=\left(2\sqrt{19}\right)^2+5^2+0^2-12^2=\mathbf{-43}$.
46.	The numbers being plugged into the function are the first five positive perfect numbers. Recall that even perfect numbers have the form $g(x)=2^{x-1}(2^x-1)$, where 2^x-1 is prime; by inspection, the five smallest positive values of x which makes this true are 2, 3, 5, 7, and 13. We have $f\big(g(x)\big)=\log_2\left(1+\sqrt{8(2^{x-1}(2^x-1))+1}\right)-2=x-1$. Thus, the answer is $(2-1)+(3-1)+(5-1)+(7-1)+(13-1)=\mathbf{25}$.
47.	Suppose $\csc x=\cot x$. This leads to $\cos x=1$ and $\sin x=0$, hence $\csc x$ would be undefined and a triangle cannot be formed. Now suppose $\sec x=\csc x$. This leads to $\cos x=\sin x=1/\sqrt{2}$, making a triangle with side lengths $\sqrt{2}$, $\sqrt{2}$, and 1. In particular, $\csc x=\sqrt{2}$. For the third case, $\sec x=\cot x$, we get the quadratic equation $\sin x=\cos^2 x\ =1-\sin^2 x$, which has the positive solution $\sin x=\frac{2}{1+\sqrt{5}}$, or $\csc x=\mathbf{\frac{1+\sqrt{5}}{2}}$. This is the largest possible value of $\csc x$.
48.	By the Power-Reducing Identities, $y=\sin^2(14x)=\frac{1-\cos(28x)}{2}$, which is a standard

	sinusoid graph of period $\frac{2\pi}{28} = \boldsymbol{\frac{\pi}{14}}$.
49.	There are three possible cases that are not necessarily disjoint: 1. 53AB 2. A53B 3. AB53 where A and B is any valid digit. Case 1 has $10 \times 10 = 100$ ways to occur, Case 2 has $9 \times 10 = 90$ ways to occur, and Case 3 has $9 \times 10 = 90$ ways to occur. Between these three cases, only Case 1 and Case 3 have a possibility of overlapping: the number 5353. By the Principle of Inclusion-Exclusion, the total number of possibilities is $100 + 90 + 90 - 1 = \mathbf{279}$.
50.	We have $\frac{A(B^2-B)\pi}{C} + D$