Question	Solution
P1.	The common ratio is $-\frac{42}{6} = \mathbf{-7}$.
P2.	We have $r^2 = 13/\pi$, so $A = \pi r^2 = \mathbf{13}$.
P3.	The odds against E is the ratio of the probability of E not happening to the probability of E happening, $\frac{1-5/6}{5/6} = \mathbf{\frac{1}{5}}$.
P4.	For acute angle θ, if $\sin\theta = 3/5$, then $\cos\theta = 4/5$, so $\sec\theta = \mathbf{5/4}$.
P5.	We have $\frac{AB}{CD} = \frac{(-7)(13)}{\left(\frac{1}{5}\right)\left(\frac{5}{4}\right)} = \mathbf{-364}$.
1.	Rewrite as $\sqrt{10-x} - \sqrt{4-x} = 2$, then square both sides and simplify to obtain $5 - x = \sqrt{(10-x)(4-x)}$. Square both sides again and simplify and get $25 - 10x + x^2 = 40 - 14x + x^2$, or $4x = 15$, thus $x = \mathbf{15/4}$.
2.	The desired amplitude is $\sqrt{2^2 + \left(-2\sqrt{3}\right)^2} = \mathbf{4}$.
3.	If $\csc x = \frac{13}{\sqrt{7}}$, then $\sin x = \frac{\sqrt{7}}{13}$ and $\cos(2x) = 1 - 2\sin^2 x = 1 - 2\left(\frac{\sqrt{7}}{13}\right)^2$, so $169\cos(2x) = 169 - 14 = \mathbf{155}$.
4.	Changing all the bases to base 2 yields $\log_2(2x) + \frac{1}{2}\log_2 x + \frac{1}{3}\log_2 x = 12$, or $\log_2\left(2x \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{3}}\right) = \log_2(2x^{\frac{11}{6}}) = 12$. In Exponential Form this is $2^{12} = 2x^{\frac{11}{6}}$, or $x = 2^6 = \mathbf{64}$.
5.	We have $AB + C + \sqrt{D} = \left(\frac{15}{4}\right)(4) + 155 + \sqrt{64} = \mathbf{178}$.

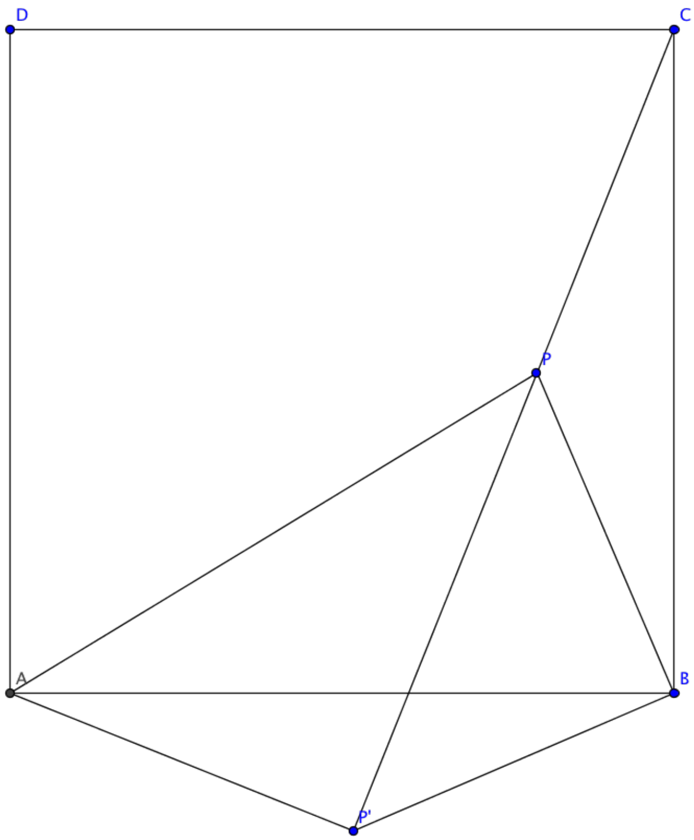
6.	Perfect squares have an odd number of positive divisors, hence it is for those values that the Tau Function will be congruent to 1 in modulo 2. The set of positive integer perfect squares less than 200 is $\{1^2, 2^2, \dots, 14^2\}$, having **14** elements.
7.	The graph of $y = f(x) = x + \sin x + e^x$ is strictly increasing. Since $f(0) = 1$ and $f(5) > 2$, we know there is one solution on the interval $x \in (0, 5)$. Because f is strictly increasing, there is exactly **1** solution.
8.	There are 91 terms inside the parentheses. Let $S = 2(\cos^2 0° + \cos^2 1° + \cos^2 2° + \cdots + \cos^2 89° + \cos^2 90°)$. By the co-function identities, $S = 2(\sin^2 90° + \sin^2 89° + \sin^2 88° + \cdots + \sin^2 1° + \sin^2 0°)$. Add the two equations to obtain $2S = 2(1 + 1 + 1 + \cdots + 1) = 2(91)$, or $S = \mathbf{91}$.
9.	By the Remainder Theorem, the desired value is simply the polynomial evaluated at $x = -1$, or $-2 - 3 + 10 + 6 = \mathbf{11}$.
10.	We have $\sqrt{A+2} + \sqrt{C + D - 2B} = \sqrt{14+2} + \sqrt{91 + 11 - 2(1)} = \mathbf{14}$.
11.	The polynomial factors as $(2x+1)(x^2-4) = 0$, so the sum of the roots is $-\mathbf{\frac{1}{2}}$. Note that this is the same value obtained via the "$-b/a$" trick, only because there are no imaginary roots.
12.	The equation simplifies to $\sin(2\theta) = 0$, or $\theta = \frac{\pi n}{2}$ for integer n. Since we have the restriction $\pi < \frac{\pi n}{2} \le 5\pi$, we must have $2 < n \le 10$. The sum of all solutions is $\sum_{n=3}^{10} \frac{\pi n}{2} = \frac{\pi}{2}(55 - 3) = \boldsymbol{26\pi}$.
13.	For $\sec B = 5/3$, we need $BC = 12$ and $AC = 9$. The area of this right triangle is **54**.

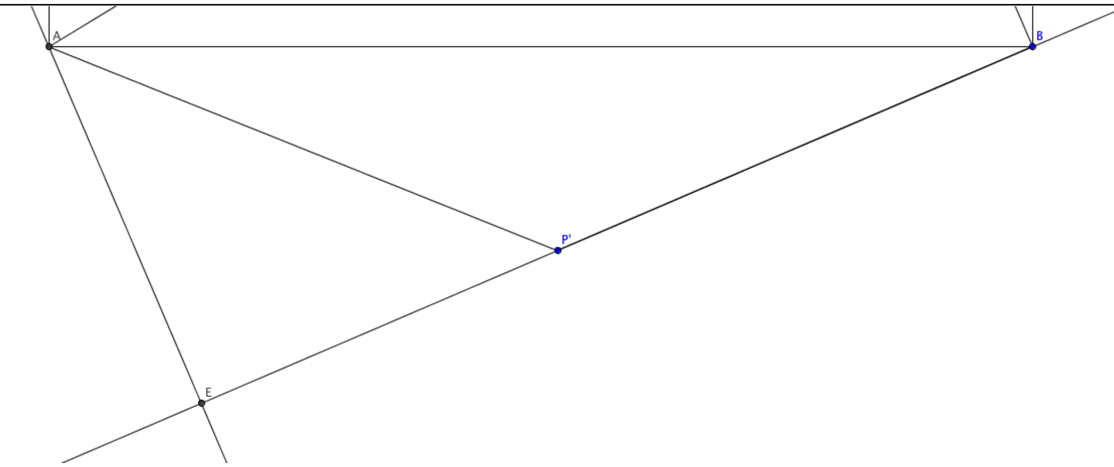
14.	The volume of a regular octahedron with edge length s is $V = \frac{\sqrt{2}}{3}s^3$. Setting this equal to 4/3 yields $s = \sqrt{2}$. The surface area of a regular octahedron with side length s is $S = 2s^2\sqrt{3}$. Thus, the answer is $2\left(\sqrt{2}\right)^2\sqrt{3} = \mathbf{4\sqrt{3}}$.
15.	We have $A\tan^2 B + CD^2 = \left(-\frac{1}{2}\right)\tan^2(26\pi) + 54\left(4\sqrt{3}\right)^2 = 0 + 54 \times 48 = \mathbf{2592}$.
16.	If $\lvert \lvert x\rvert - 7\rvert \leq 8$, then $-8 \leq \lvert x\rvert - 7 \leq 8$, or $\lvert x\rvert \leq 15$, or $-15 \leq x \leq 15$ since the absolute value of any number must be at least 0. There are **31** integers in this interval.
17.	The slope of the line needs to equal $\tan 30° = 1/\sqrt{3}$. From the given equation, the slope of the line is $2/k$. Thus, $k = 2\sqrt{3}$, or $k^4 = \mathbf{144}$.
18.	The maximum value will be achieved when the triangle is equilateral. Thus, $\sin A \sin B \sin C = \left(\frac{\sqrt{3}}{2}\right)^3 = M$ and $128M^2 = 128\left(\frac{27}{64}\right) = \mathbf{54}$.
19.	There are six complex solutions to the equation, consisting of three conjugate pairs. The "principal" solution, $2e^{\frac{\pi}{6}i}$, has a positive real part and therefore, so will its conjugate. The product of two complex conjugates is the norm-square, so the answer is $\left\lvert 2e^{\frac{\pi}{6}i}\right\rvert^2 = 2^2 = \mathbf{4}$.
20.	We have $A + \sqrt{B} + \frac{2C}{D} = 31 + \sqrt{144} + \frac{2(54)}{4} = \mathbf{70}$.
21.	Notice that $39^2 + 52^2 = 25^2 + 60^2$. Thus, the quadrilateral in question consists of two right triangles glued together at their hypotenuse. The area is therefore equal to $\frac{(39)(52)+(25)(60)}{2} = \mathbf{1764}$.

22.	The side length of the cube is $\cos x$. We have $6(\cos x)^2 = 36/17$, or $\cos^2 x = \frac{6}{17}$, so $\sin^2 x = 1 - \frac{6}{17} = \frac{11}{17} = \frac{m}{n}$, making $m + n = \mathbf{28}$.
23.	Since $6912 = 19(360) + 72$, the answer is **72** degrees.
24.	Guess-and-check yields $M = 512 = 8^3$ and $N = 343 = 7^3$, so the answer is **15**.
25.	We have $A - B + C - D^2 = 1764 - 28 + 72 - 15^2 = \mathbf{1583}$.
26.	Observe that M is a sort of permutation-scaling matrix, where the first element goes to the fourth position and gets scaled by $\frac{1}{4}$, the second element goes to the first position without scaling, etc. Using this reasoning, we can go backwards and deduce that $M^{-1} \times \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 4d \\ a \\ 2b \\ \frac{c}{3} \end{bmatrix}$, making $M^{-1} = \begin{bmatrix} 0 & 0 & 0 & 4 \\ 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 \end{bmatrix}$, so the sum of the elements of $3M^{-1}$ is $3\left(4 + 1 + 2 + \frac{1}{3}\right) = \mathbf{22}$.
27.	The given function can be written as $f(x) = 1 - \frac{3}{4}\sin^2 x$, having maximum value of 1 and minimum value of $\frac{1}{4}$. The midpoint of I is $\mathbf{\frac{5}{8}}$.
28.	Since the θ coefficient is even, the number of petals is $2 \times 24 = \mathbf{48}$.
29.	The parabola in standard form is $-8(x+2) = (y-3)^2$, so the vertex is at $(-2, 3)$ and $p = \left\lvert -\frac{8}{4} \right\rvert = 2$. Since this is a left-facing parabola, the coordinates of the focus is $(-2-2, 3) = (-4, 3)$, and its distance from the origin is $\sqrt{(-4)^2 + 3^2} = \mathbf{5}$.
30.	We have $\left(\frac{CB}{D}\right)^A = \left(\frac{\left(\frac{5}{8}\right)(48)}{5}\right)^{22} = 6^{22}$, which always has **6** for a units digit.

31.	Suppose P has degree n. The left-hand side of the equation has degree $2n$ while the right-hand side has degree of $n+1$. Therefore, $n=1$ and P is a linear function, say $P(x)=mx+b$. Substitute this into the equation to obtain $mx^2+b+2x^2+10x=2x(m(x+1)+b)+3$, and combine like-coefficients to get $(m+2)x^2+10x+b=2mx^2+(2b+2m)x+3$. Setting corresponding coefficients to each other yields $m=2$ and $b=3$, so $P(x)=2x+3$ and $P(100)=2(100)+3=\mathbf{203}$.
32.	From the Extended Law of Sines, $\frac{\sin\alpha}{a}=\frac{\sin\beta}{b}=\frac{\sin\gamma}{c}=\frac{1}{2R}$. Adding up these three equations yields $\sin\alpha+\sin\beta+\sin\gamma=\frac{a+b+c}{2R}=\frac{p}{2R}=\frac{5}{2(1)}=\mathbf{\frac{5}{2}}$. (Note: Inscribed triangles in the unit circle with a perimeter of 5 *is* possible. For example, pick one vertex to be at (1,0), the second at (0,1), and the third to be near the point $\left(\cos\frac{5\pi}{4},\sin\frac{5\pi}{4}\right)$. Such a triangle will have perimeter greater than 5. By continuity, a perimeter of exactly 5 is attainable.)
33.	All arguments are in degrees. By the Sum-to-Product and Co-Function Identities, $\sin 40+\sin 20=2\sin\frac{40+20}{2}\cos\frac{40-20}{2}=2\sin 30\cos 10=\cos 10=\sin 80$. Taking the inverse sine of this expression yields **80**.
34.	Note that $a_{32}=16+(16\times 2)^2=16(1+(64))=(16)(65)$ and $a_{33}=16+(2\times 16+1)^2=4(16)^2+16+4(16)+1=(16+1)(4\times 16+1)=(17)(65)$, so we suspect that **65** is the desired maximum GCD. Note that $(3+2n)a_n+(1-2n)a_{n+1}=65$ for all n; therefore, any GCD of a_n and a_{n+1} will be a multiple of 65. Thus, 65 is the maximum GCD indeed.
35.	We have $A+BC+D=203+\left(\frac{5}{2}\right)(80)+65=\mathbf{468}$.
36.	Let P equal the intersection of the medians BE and AD. Point P divides the medians into a 2:1 ratio, so AP = 4 and EP = 3. Triangle APE is a right triangle

	with area 6, which happens to be one-sixth the area of ABC. The answer is **36.**
37.	The slope of the segment connecting the endpoints is 1; therefore, θ must be coterminal to $\frac{\pi}{4}$, or $\theta = \frac{\pi}{4} + 2\pi n$ for integer n. The r-values for each endpoint are $(\sqrt{2})\sqrt{2} = 2$ and $(64\sqrt{2})\sqrt{2} = 128$. Thus, we have $2^1 \leq 2^{\frac{2\theta}{\pi}} \leq 2^7$, or $\frac{\pi}{2} \leq \theta \leq \frac{7\pi}{2}$. Therefore, $\frac{\pi}{2} \leq \frac{\pi}{4} + 2\pi n \leq \frac{7\pi}{2}$, leading to a singular answer of $n = 1$. There is only **1** intersection point.
38.	Let θ denote the in-between angle. The desired ratio is equal to $\frac{\frac{1}{2}(8)(15)\sin(\theta)}{\frac{1}{2}(8)(15)\sin 2\theta} = \frac{\sin\theta}{2\sin\theta\cos\theta} = \frac{\sec\theta}{2}$. If $\sin\theta = 8/17$, then $\cos\theta = 15/17$, so $\frac{\sec\theta}{2} = \frac{\frac{17}{15}}{2} = \mathbf{17/30}$.
39.	The first few terms of the sequence are $a_1 = 2$, $a_2 = 1 - \frac{1}{a_1} = 1 - \frac{1}{2} = \frac{1}{2}$, $a_3 = 1 - \frac{1}{a_2} = 1 - \frac{1}{\frac{1}{2}} = -1$, and this cycle continues onward. Every three terms starting from the first will have a sum of 1.5, and since $833 = 3(277) + 2$, the desired sum is $277(1.5) + 2 + \frac{1}{2} = \mathbf{418}$.
40.	We have $A + \frac{D}{30BC+2} = 36 + \frac{418}{30\left(\frac{17}{30}\right)+2} = 36 + 22 = \mathbf{58}$.
41.	If $S = \{1\}$, then $\sum_{n=1}^{1} \frac{1}{\Pi(S_n)} = 1$. If $S = \{1, 2\}$, then $\sum_{n=1}^{3} \frac{1}{\Pi(S_n)} = 2$. In general, it can be proven by induction that if $S = \{1, 2, 3, \dots, n\}$, then $\sum_{n=1}^{2^n-1} \frac{1}{\Pi(S_n)} = n$, so for this problem the answer is **4.**
42.	Let θ denote the angle opposite the side with length a. We have $\sin(2\theta) = 2\sin\theta\cos\theta = 2\left(\frac{a}{2\sqrt{ab}}\right)\left(\frac{b}{2\sqrt{ab}}\right) = \frac{1}{2}$. Thus, $2\theta = 30°$ or $\theta = \mathbf{15°}$.

43.	We have $12\sin(-u) - .5\cos(-v)\tan(-w) = -12\sin u + .5\cos v \tan w =$ $-12\left(\frac{3}{4}\right) + .5\left(-\frac{1}{7}\right)(28) = \mathbf{-11}$.
44.	 Refer to the diagram above. Take triangle CPB and rotate it across B so that C coincides with A, resulting in triangle AP'B. We have PB = P'B and $m\angle$CBP = $m\angle$ABP', so triangle PBP' is a 45-45-90 triangle, making $PP' = (2\sqrt{2})\sqrt{2} = 4$. Consequently, triangle APP' is a 3-4-5 right triangle, where ∠PP'A is the right angle. To obtain the area of square ABCD, we only need to find the value of AB^2. Extend segment P'B into a line and drop altitude AE, as shown below.

	 We have $\mathrm{CP} = \mathrm{AP}' = 3$ and since $AP'E$ is a 45-45-90 triangle, $AE = EP' = 3/\sqrt{2}$. Thus, by the Pythagorean Theorem, $AB^2 = \mathrm{AE}^2 + \mathrm{EB}^2 = \left(\frac{3}{\sqrt{2}}\right)^2 + \left(\frac{3}{\sqrt{2}} + 2\sqrt{2}\right)^2 =$ **29**.
45.	We have $10D - A^{B+C} = 10(29) - (4)^{15-11} = 290 - 256 = \mathbf{34}$.
46.	Since $2(3n+5) - 3(2n+3) = 1$, $3n+5$ and $2n+3$ are relatively prime. Also, since $2210 = 2 \times 5 \times 13 \times 17$, we have $\frac{2210}{(3n+5)(2n+3)} = \frac{2\times5\times13\times17}{(3n+5)(2n+3)} = 5\frac{(2\times13)(17)}{(3n+5)(2n+3)}$. Setting $3n+5 = 26$ and $2n+3 = 17$ yields $n = \mathbf{7}$. Turns out this is the only valid value for n.
47.	By the Sum-and-Difference Formulae, the left-hand-side of the equation simplifies to $\sin\frac{2x+4x}{3}\cos\frac{16x-6x}{5} = \sin(2x)\cos(2x)$, or $\frac{1}{2}\sin(4x)$ by the Double-Angle Formula for sine. Thus, the equation is $\frac{1}{2}\sin(4x) = \frac{1}{4}$, and $4x = \frac{\pi}{6}$, making $x = \boldsymbol{\pi/24}$.
48.	Note that $M = \begin{bmatrix} 1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 \end{bmatrix} = \begin{bmatrix} \cos\frac{-\pi}{3} & -\sin\frac{-\pi}{3} \\ \sin\frac{-\pi}{3} & \cos\frac{-\pi}{3} \end{bmatrix}$, a clockwise rotation of $\frac{\pi}{3}$

	about the origin. Thus, $M^{2013} = \begin{bmatrix} \cos\frac{-2013\pi}{3} & -\sin\frac{-2013\pi}{3} \\ \sin\frac{-2013\pi}{3} & \cos\frac{-2013\pi}{3} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$. The sum of the squares of the elements is **2**.
49.	Since $25^2 < 650 < 26^2$, f will take on every integer from 1 to 25, inclusive. We find that f equals 1 for two values (1 and 2), f equals 2 for four values (3, 4, 5, and 6), f equals 3 for six values, etc. Based on this pattern, we have $\sum_{n=1}^{650} \frac{1}{f(n)} = 2(1) + 4\left(\frac{1}{2}\right) + 6\left(\frac{1}{3}\right) + \cdots 50\left(\frac{1}{25}\right) = 2 + 2 + 2 + \cdots + 2 = 25(2) = \mathbf{50}$.
50.	We have $AC + \frac{\pi}{B} + D = (7)(2) + \frac{\pi}{\pi/24} + 50 = \mathbf{88}$.