Note: For all questions, answer "(E) NOTA" means none of the above answers is correct.

1. What is the 197th smallest triangular number?

   (A) 19503 (B) 19403 (C) 19501 (D) 19504 (E) NOTA

2. In triangle ABC, $m\angle ABC = 90°$, $AB = 12$, $AC = 20$, and D is a point on $\overline{BC}$ such that $\overline{AD}$ bisects $\angle BAC$. Find AD.

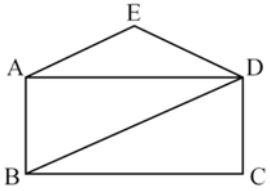
   (A) $6\sqrt{5}$ (B) $7\sqrt{5}$ (C) $12\sqrt{3}$ (D) $10\sqrt{3}$ (E) NOTA

3. Triangle ABC is isosceles, where AB = AC. Let D be the foot of the altitude to segment BC from vertex A. Which of the following is necessarily true?

   I. Segment AD is a median.
   II. Segment AD is a perpendicular bisector.
   III. Segment AD is an angle bisector.
   IV. Segment AD is a cevian.

   (A) I (B) I, II (C) I, II, III (D) I, II, III, IV (E) NOTA

4. In the figure below, ABCD is a rectangle and ABDE is an isosceles trapezoid, where $\overleftrightarrow{AE}||\overleftrightarrow{BD}$ and AB = ED. If AE = 6 and BD = 12, find the length of segment DC.



   (A) $8\sqrt{2}$ (B) 8 (C) $6\sqrt{2}$ (D) 6 (E) NOTA

5. In convex quadrilateral $ABCD$, angles A and C are right angles, and angle B has a measure of 75°. If $m\angle ADB = 60°$ and $BD = 12\sqrt{3}$, find the perimeter of the quadrilateral.

   (A) $18 + 6\sqrt{3} + 12\sqrt{6}$ (B) $10 + 8\sqrt{2} + 9\sqrt{6}$

   (B) $16 + 5\sqrt{6} + 12\sqrt{3}$ (D) $12 + 3\sqrt{3} + 18\sqrt{6}$ (E) NOTA

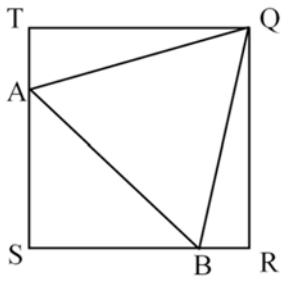
6. Let T_1 be an isosceles right triangle with a leg of length 4. Triangle T_2 is formed by connecting the midpoints of the sides of T_1. In general, for integer $n > 1$, triangle T_n has vertices that consist of the midpoints of the sides of T_{n-1}. Let $P(T)$ denote the perimeter of triangle T. Evaluate: $\sum_{n=1}^{\infty} P(T_n)$

   (A) $12\sqrt{2} + 4$ (B) 32 (C) $16 + 8\sqrt{2}$ (D) $24 - 6\sqrt{2}$ (E) NOTA

7. Let θ be an acute angle, and x and y be positive numbers. If $x \sec\theta = \sqrt{x^2 + 9y^2}$, find the value of $\cot\theta$ in terms of x and y.

   (A) $\frac{3x}{y}$ (B) $\frac{x}{3y}$ (C) $\frac{y}{3x}$ (D) $\frac{3y}{x}$ (E) NOTA

8. In the figure below, QRST is a square having side length 4, and ABQ is an equilateral triangle with points A and B lying on segments TS and SR, respectively. Find the length of segment AQ.



   (A) $4\sqrt{2} + 4\sqrt{6}$ (B) $\frac{8}{\sqrt{6}-\sqrt{2}}$ (C) $4\sqrt{3}$ (D) 2 (E) NOTA

9. Select from the following the best item to fill in the blank to complete the statement of *Pascal's Theorem*: If a hexagon is inscribed in a conic section, then the points of intersection of the pairs of opposite sides (extending them, if necessary) of the hexagon are _____.

   (A) vertices of an equilateral triangle (B) concurrent

   (C) collinear (D) platonic (E) NOTA

10. Triangle JKL has side lengths JL = 8, KL = 15, and JK = 17. Let X be the foot of the altitude from vertex L. Find the value of $\sin\angle KLX + \csc\angle JLX$.

    (A) 369/136 (B) 379/136 (C) 379/149 (D) 369/146 (E) NOTA

11. In triangle ABC, AB = 1, $m\angle C = 37°$, and $m\angle A = 75°$. Find BC.

(A) $\frac{\cos 75°}{\cos 37°}$ (B) $\frac{\sin 37°}{\sin 75°}$ (C) $\sin 38°$ (D) $\frac{\sin 68°}{\cos 35°}$ (E) NOTA

12. A triangle is inscribed in a circle with a radius of length 6. One angle in this triangle has a measure of 45°, while another angle in this triangle has a measure of 105°. Find the perimeter of this triangle.

(A) $\frac{5\sqrt{2}+4+3\sqrt{6}}{2}$ (B) $\frac{5\sqrt{2}+4+3\sqrt{3}}{2}$ (C) $\frac{5\sqrt{2}+2+3\sqrt{6}}{2}$ (D) $\frac{5\sqrt{3}+4+3\sqrt{6}}{2}$ (E) NOTA

13. A triangle has one side of length 12. Opposite this side is an angle with a measure of $\pi/6$ radians. Find the area of the circumcircle of this triangle.

(A) 288π (B) 144π (C) 72π (D) 36π (E) NOTA

14. Select from the following the best item to fill in the blank to complete the statement of *Morley's Theorem*: In a triangle, the three points of intersection of the adjacent angle trisectors are _____.

(A) vertices of an equilateral triangle (B) concurrent

(C) collinear (D) platonic (E) NOTA

15. A Samoyed mother leaves her dog house and travels 37 miles south, 27 miles east, 70 miles north, and 17 miles east at a constant velocity of 5 miles per hour to reach the point X. The Samoyed's puppy leaves 30 minutes after the mother leaves in order to search for her mother, heading directly to point X along a straight road. The puppy needs to move m/n miles per hour in order to meet her mother at point X at the same time, where m and n are relatively prime positive integers. Find the value of $m + n$.

(A) 77 (B) 512 (C) 847 (D) 1579 (E) NOTA

16. What is the maximum possible area of a triangle with two sides of length 8 and $8\sqrt{6}$?

(A) $64\sqrt{3}$ (B) $32\sqrt{3}$ (C) $64\sqrt{6}$ (D) $32\sqrt{6}$ (E) NOTA

17. Triangle ABC is a right triangle, with $\angle B$ the right angle, with $AB = c$, $BC = a$, and $AC = b$. Let $\theta = \angle A$. Express $\sin\frac{\theta}{2}$ in terms of a, b, and c.

(A) $\sqrt{\frac{b-c}{2b}}$ (B) $\sqrt{\frac{b+c}{2b}}$ (C) $\frac{1}{2}\sqrt{\frac{b-c}{b}}$ (D) $\frac{a}{2}$ (E) NOTA

18. How many triangles ABC exist such that BC = 20, AB = 16, and $m\angle A = 30°$?

(A) 0 (B) 1 (C) 2 (D) Infinitely many. (E) NOTA

19. Find AB in triangle ABC if $m\angle A = 67°$, $m\angle B = 56°$, and AC = 15.

(A) $\frac{\cos 57°}{15\cos 56°}$ (B) $\frac{15\sin 123°}{\cos 34°}$ (C) $15\cos 126°$ (D) $15\frac{\sin 57}{\sin 56°}$ (E) NOTA

20. Find the area of a triangle with side lengths 26, 35, and 51.

(A) 420 (B) 390 (C) 360 (D) 264 (E) NOTA

21. Let ABC be an acute, scalene triangle. Denote the following points as follows:

- p_1, p_2, and p_3 are the foot of the medians of ABC.
- p_4, p_5, and p_6 are the foot of the altitudes of ABC.
- p_7, p_8, and p_9 are the foot of the angle bisectors of ABC.

A point p is chosen at random from the set $S = \{p_i | i \text{ is a positive integer less than } 10\}$, where each element of S is equally likely to be chosen. Find the probability that p lies along the circumference of the nine-point circle (also known as Feuerbach's Circle) of ABC.

(A) 2/3 (B) 1/3 (C) 0 (D) 1 (E) NOTA

22. Triangle EFG is isosceles, where EF = EG = 10 and FG = 8. Let H be a point on segment FG such that FH = 2. If EH = x, find the value of x^2.

(A) 22 (B) 44 (C) 66 (D) 88 (E) NOTA

23. In triangle PQR, PQ = 8 and QR = 15. Let S, T, and U be points on sides $\overline{QP}$, $\overline{PR}$, and $\overline{QR}$, respectively, such that PS = 6, UR = 5, and the segments $\overline{QT}$, $\overline{PU}$, and $\overline{RS}$ are all concurrent. The ratio of the area of triangle QTR to the area of triangle QTP is equal to m/n, where m and n are relatively prime positive integers. Find $m + n$.

(A) 8 (B) 7 (C) 6 (D) 5 (E) NOTA

24. Triangle ABC is isosceles, where AB = AC = 1. The foot of the angle bisector of ∠ABC is at D. Furthermore, AD + DB = BC. Find the measure of angle BAC.

(A) 120° (B) 150° (C) 75° (D) 100° (E) NOTA

25. Triangle ABC has AC = 4, $m\angle B = 45°$, and $m\angle A = 60°$. Points D, E, and F are chosen along sides $\overline{AB}$, $\overline{AC}$, and $\overline{BC}$, respectively, such that the perimeter of triangle DEF is as small as possible. Find the ratio of the area of triangle DEF to the area of triangle ABC.

(A) $\frac{\sqrt{5}}{2} - \frac{1}{2}$ (B) $\frac{\sqrt{3}}{4} - \frac{1}{4}$ (C) $\sqrt{\frac{2}{3}}$ (D) $\frac{\sqrt{3}-\sqrt{2}}{2}$ (E) NOTA

26. Let α, β, and γ be the angles of a triangle. Find the maximum value of the multivariable function given by $F(\alpha, \beta, \gamma) = \frac{\tan\alpha + \tan\beta + \tan\gamma}{\tan\alpha tan\beta \tan\gamma}$.

(A) 2.00 (B) 1.50 (C) 1.00 (D) 0.50 (E) NOTA

27. Let L equal the side length of the smallest equilateral triangle in which three discs of radii length 2, 3, and 4 can be placed without overlap. Find the sum of the digits of L^2 when expressed in base 10.

(A) 9 (B) 12 (C) 15 (D) 18 (E) NOTA

28. In triangle ABC, the sides opposite angles $\angle A$, $\angle B$, and $\angle C$ have lengths of a, b, and c, respectively. If $a = 986$, $b = 4523$, and $c = 5100$, find the value of $b\cos\angle A + a\cos\angle B$.

(A) 5509 (B) 4523 (C) 5100 (D) 6086 (E) NOTA

29. Let x, y, and z be positive real numbers such that

$$x^2 + y^2 = 7$$

$$x^2 + z^2 + xz = 21$$

$$y^2 + z^2 + yz\sqrt{3} = 28.$$

Find the value of $2xy + xz\sqrt{3} + yz$.

(A) 49 (B) $2\sqrt{14}$ (C) $7\sqrt{6}$ (D) $14\sqrt{3}$ (E) NOTA

30. The course of a derby race is made up of a straight downhill run and a 150-meter level portion. When the starting point of the race is spotted from the finish line, the angle of elevation is 30 degrees. Also, the angle the hill makes with the ground is 45 degrees. Find the length, in meters, of the downhill run.

(A) $75 + 25\sqrt{3}$ (B) $\frac{30\sqrt{3}}{\sqrt{3}-\sqrt{2}}$ (C) 150 (D) $75\sqrt{2} + 75\sqrt{6}$ (E) NOTA