Note: For all questions, answer "(E) NOTA" means none of the above answers is correct.

1. Evaluate: $\cot\frac{\pi}{6}$

(A) 0 (B) $\frac{\sqrt{3}}{3}$ (C) $\sqrt{3}$ (D) 1 (E) NOTA

2. How many petals does the polar graph $r=43\cos(2013\theta)$ have as θ ranges through the real numbers?

(A) 4026 (B) 4025 (C) 2013 (D) 2012 (E) NOTA

3. Which of the following angles, in radians, is equivalent to 36 arcseconds?

(A) $\frac{\pi}{36}$ (B) $\frac{\pi}{18000}$ (C) $\frac{\pi}{3600}$ (D) $\frac{\pi}{1800}$ (E) NOTA

4. Express the following in terms of cosine functions: $\sin\left(\arctan\left(\cos x\right)\right)$.

(A) $\frac{\sqrt{1+\cos^2 x}}{\cos x}$ (B) $\sqrt{1+\cos^2 x}$ (C) $-\frac{\cos x}{\sqrt{1+\cos^2 x}}$ (D) $\frac{\cos x}{\sqrt{1+\cos^2 x}}$ (E) NOTA

5. Which of the following trigonometric functions is odd for all values of x?

(A) $f(x)=\cos(3x)$ (B) $g(x)=\sin(2x)$

(C) $h(x)=\tan^2(x)$ (D) $y(x)=\sec^3(x)$ (E) NOTA

6. Two rays form an angle in the Cartesian Plane, with vertex at the origin. If one of the rays is the positive x-axis, the other ray points towards Quadrant III, and the tangent of the resulting angle is $\frac{12}{5}$, what is the cosine of this angle?

(A) $-\frac{5}{13}$ (B) $\frac{5}{13}$ (C) $-\frac{12}{13}$ (D) $\frac{12}{13}$ (E) NOTA

7. What is the sum of the solutions of the equation $\tan\theta=\sqrt{3}$, where $-2\pi<\theta<2\pi$?

(A) $\frac{\pi}{3}$ (B) $\frac{5\pi}{3}$ (C) 0 (D) $-\frac{2\pi}{3}$ (E) NOTA

8. The sides of a triangle have lengths of 65, 72, and 97. This triangle is:

(A) Acute (B) Obtuse (C) Right (D) Degenerate (E) NOTA

9. Restricted to $\theta \in [0, 2\pi)$, what is the argument of $3 + i\sqrt{3}$? Note that here, $i = \sqrt{-1}$.

(A) $\frac{\pi}{3}$ (B) $-\frac{\pi}{3}$ (C) $\frac{\pi}{6}$ (D) $\frac{7\pi}{6}$ (E) NOTA

10. What is the cosine of the angle between the complex vectors $1+2i$ and $2-3i$? Note that here, $i = \sqrt{-1}$, and ***not*** the standard horizontal unit vector.

(A) $-\frac{4\sqrt{65}}{65}$ (B) $\frac{4\sqrt{65}}{65}$ (C) $-\frac{\sqrt{65}}{4}$ (D) $\frac{\sqrt{65}}{4}$ (E) NOTA

11. A helix is parameterized by $x = 4\cos t$, $y = 3\sin t$, and $z = t$. What is the distance between the points in the helix at time $t = 0$ and $t = \pi$?

(A) $\sqrt{16+\pi^2}$ (B) $\sqrt{36+\pi^2}$ (C) $\sqrt{64+\pi^2}$ (D) $4+\pi$ (E) NOTA

12. Express the line $3x + 5y = 7$ in polar form.

(A) $r = \frac{7}{3\cos\theta + 5\sin\theta}$ (B) $r = \frac{7}{3\sin\theta + 5\cos\theta}$ (C) $r = \tan^{-1}(3\cos\theta + 5\sin\theta)$

(D) $r = \frac{-7}{3\cos\theta - 5\sin\theta}$ (E) NOTA

13. Evaluate the sum $\sum_{n=0}^{\infty} 2\cos^n(2\theta)$, for $0 < \theta < \pi/2$.

(A) $\cot\theta$ (B) $\csc\theta\sec\theta$ (C) $\sec^2\theta$ (D) $\csc^2\theta$ (E) NOTA

14. For real number *x*, what is the maximum value of the function: $f(x) = \cos^4 x - \sin^2 x$?

(A) $\frac{1}{2}$ (B) 1 (C) $\frac{\sqrt{3}}{6}$ (D) $\frac{\sqrt{2}}{2}$ (E) NOTA

15. What is the period of the graph of $y = \sin^5 x + \sin^2 x$?

(A) 15π (B) 5π (C) 2π (D) $\frac{\pi}{15}$ (E) NOTA

16. One of the first trigonometric functions is the Chord Function, defined as: $\text{crd}(\theta) := 2\sin\frac{\theta}{2}$. What is $\cos^2\theta$ in terms of the Chord Function?

(A) $\frac{\text{crd}^2(\theta)}{2}$ (B) $\frac{4-\text{crd}^2(\theta)}{4}$ (C) $1+4\text{crd}(\theta)+4\text{crd}^2(\theta)$ (D) $1-4\text{crd}(\theta)+4\text{crd}^2(\theta)$ (E) NOTA

17. What is the entry in the first row, first column of the matrix M^2, if $M=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$?

(A) $1-2\sin^2\theta$ (B) $\sin\theta\cos\theta$ (C) 1 (D) $\sin(2\theta)$ (E) NOTA

18. The value of $\sec\frac{\pi}{12}$ can be expressed in the form of $a\sqrt{\sqrt{b}+c\sqrt{d}}$ where a,b,c,d are integers, *a* is even, and *d* is square-free. What is the product $abcd$?

(A) 16 (B) −12 (C) −24 (D) −36 (E) NOTA

19. What is the value of $\sum_{n=2}^{6} e^{\left(\frac{n}{2}\right)\pi i}$? Note that here, $i=\sqrt{-1}$.

(A) 2 (B) 1 (C) 0 (D) −1 (E) NOTA

20. Express $\tan 75°$ in terms of radicals.

(A) $\sqrt{6}+\sqrt{3}$ (B) $2+\sqrt{3}$ (C) $2-\sqrt{3}$ (D) $\sqrt{6}-\sqrt{3}$ (E) NOTA

21. Triangle *XYZ* has side lengths of $x=5$, $y=6$, and $z=4$. What is the value of $\tan\frac{Y}{2}$? Recall that *x*, *y*, and *z* represent the lengths of sides *YZ*, *XZ*, and *XY*, respectively.

(A) $\frac{\sqrt{7}}{3}$ (B) $\frac{\sqrt{7}}{2}$ (C) $\frac{3\sqrt{7}}{2}$ (D) $\frac{\sqrt{7}}{4}$ (E) NOTA

22. What is the range of $f(x)=\cot^{-1}x$?

(A) $[0,2\pi)$ (B) $[-\pi,\pi]$ (C) $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ (D) $(-\infty,\infty)$ (E) NOTA

23. What is the probability that a randomly chosen angle $\theta \in [0, 2\pi)$ satisfies $\cos(2\theta) < \sin\theta$? Assume a uniform distribution of angles.

(A) $\frac{3}{4}$ (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{1}{6}$ (E) NOTA

24. Solve for smallest positive value of *x* such that $\cos(3x) + 3\cos x - 4 = 0$.

(A) 0 (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$ (E) NOTA

25. How many distinct obtuse triangles can be made by selecting three distinct integers between 1 and 10, inclusive, as side lengths?

(A) 6 (B) 10 (C) 12 (D) 15 (E) NOTA

26. Given the triangle *ABC*, where $a = 4$, $m\angle A = 75°$, and $m\angle B = 60°$, find the area of the triangle. Note that *a* denotes the length of side *BC*.

(A) $16 + 4\sqrt{3}$ (B) $8\sqrt{3} - 4$ (C) $12 - 4\sqrt{3}$ (D) $2\sqrt{3} + 4$ (E) NOTA

27. Which of the following expressions has the smallest numerical value?

(A) $\cos 1$ (B) $\sin 1$ (C) $\tan 1$ (D) $\cot 1$ (E) NOTA

28. Which of the following expressions is equivalent to $\cos x + \sin x$ for all values of *x*?

(A) $\sqrt{2}\cos\left(\frac{\pi}{4} + x\right)$ (B) $\sqrt{2}\sin\left(\frac{\pi}{4} - x\right)$ (C) $\frac{\sqrt{2}}{2}\tan\left(\frac{\pi}{8} + x\right)$ (D) $\frac{\sqrt{2}}{2}\sin\left(\frac{\pi}{4} + x\right)$ (E) NOTA

29. Solve for *x*: $\cos^{-1} x = \sin^{-1}(3x)$

(A) $\frac{\sqrt{10}}{10}$ (B) $\frac{\sqrt{5}}{5}$ (C) $-\frac{\sqrt{10}}{10}$ (D) $-\frac{\sqrt{10}}{5}$ (E) NOTA

30. Which of the following polar coordinates (r, θ) is equivalent to the rectangular coordinates $(x, y) = \left(\sqrt{5} + 1, \sqrt{10 - 2\sqrt{5}}\right)$?

(A) $\left(4, 2\pi/5\right)$ (B) $\left(4, \pi/5\right)$ (C) $\left(4, -\pi/5\right)$ (D) $\left(4, -2\pi/5\right)$ (E) NOTA