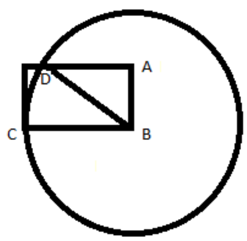
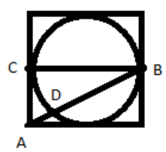
1. **B**—$116 \cdot 124 = (120-4)(120+4) = 120^2 - 4^2 = 14400 - 16 = 14384$
2. **D**—Sum of interior angles is 180(n-2); sum of exterior angles is 360; ratio is $\frac{n-2}{2}$. Solving $\frac{n-2}{2} = 17$ gives 36.
3. **B**—We have $y^2 - x^2 = 11^2 \rightarrow (y-x)(y+x) = 121$; since 121's only factors are 1, 11, and 121, we must have y – x = 1 and y+x = 121. Hence y = 61, x = 60; 6*1 = 6.
4. **B**—Write $\frac{1}{k(k+1)} = \frac{1}{k} - \frac{1}{k+1}$; then our sum becomes $1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \cdots + \frac{1}{2013} - \frac{1}{2014}$ $= 1 - \frac{1}{2014} = \frac{2013}{2014}$.
5. **A**—9999 = 10000 – 1 = $100^2 - 1^2$ = 99(101) ; 101 is prime; 1+0+1 = 2.
6. **E**—The probability that our numbers are equal is $\frac{1}{15}$; hence, the probability that our numbers are different is $\frac{14}{15}$. Since both numbers are randomly chosen, the probability that my number is higher is half of this, or $\frac{7}{15}$.
7. **D**—The sum is $\frac{2013 \cdot 2014}{2} = 2013 \cdot 1007 = (2000+13) \cdot (1000+7)$. If we expand this, only the $13 \cdot 7$ term will matter (we don't have to compute the rest); $13 \cdot 7 = 91$.
8. **B**—The product of the LCM and GCD of two numbers is the product of the numbers.
9. **D**—If you were to do this the way you were taught in elementary school, you'd have one 1 in the ones column, two 1s in the tens column, three in the 100s column,…up to nine 1s, at which point you then have 8, 7, 6,…, 1. So the answer is just 1+2+3+…+9+8+7+…+1 = 8(9)/2 + 9 + 8(9)/2 = 81.
10. **C**—The sum of the reciprocals of the factors is the sum of the factors divided by the number itself (can you figure out why?). Hence, this is 72/30 = 12/5.
11. **B**—The die are fair; the minimum sum is 4*1= 4*;* the maximum sum is 4*12=48. Since these can be formed in only one way, these have minimal probability. The sums occurring with maximal probability are those which can be formed in the most ways using four not necessarily distinct integers between 1 and 12; this is the "middle" number, or ½ (4 + 48) = 26.
12. **B**—The probability of the sum being greater than 5 is 1 minus the probability the sum is 5 or less. There are 4 ways to roll the dice and get a sum of 5; rolling (1, 4), (2, 3), (3, 2), and (2, 1). There are 3 ways to get a sum of 4: (1, 3), (2, 2), (3, 1). There are 2 ways to get a sum of 3, and 1 way to get a sum of 2. There are 12*12 = 144 total possibilities. Hence the probability of the sum being greater than 5 is $1 - \frac{10}{144} = \frac{67}{72}$.
13. **B**—If a+b=c+d, then we'd have 16a+4b=16c+4d and a+b=c+d; subtract 4 times the second equation from the first to get 12a =12c → a=c → b=d. However, this means that P(x) =Q(x) for all x, which violates the strict inequality. (Choice A can happen if a is positive; C and D can happen if c is negative)
14. **C**—L_2 and L_3 intersect at the point (3, 6); so we know that 6 = $3\alpha + \beta \rightarrow \beta = 6 - 3\alpha$. Hence, this line intersects $y = 2$ for some x such that $\alpha x + \beta = 2 \rightarrow x = 3 - \frac{4}{\alpha}$. Since L_3 intersects the line at (1, 2), the triangle has height 4 and base $\left(3 - \frac{4}{\alpha}\right) - 1) = 2 - \frac{4}{\alpha}$; its area is therefore $\frac{1}{2}(4)\left(2 - \frac{4}{\alpha}\right) = 4 - \frac{8}{\alpha} = \frac{4\alpha - 8}{\alpha}$.

15. **C**—$\frac{n^2+16}{n-4} = \frac{n^2-16}{n-4} + \frac{32}{n-4} = n + 4 + \frac{32}{n-4}$; since n+4 is always an integer, we just need the sum of all n > 4 such that $\frac{32}{n-4}$ is an integer. The factors of 32 are 1, 2, 4, 8, 16, 32; so we set n – 4 equal to each of these in turn, or, to take a shortcut, we compute the sum of the factors and add 4 times the number of factors. This is 2^6 – 1 + 4*6 = 87.
16. **D**—If the polynomial is $a_1x^5 + a_2x^4 + a_3x^3 + a_4x^2 + a_5x + a_6$, there are six coefficients to solve for, so we need six equations; we can get six equations by plugging in six different points.
17. **C**—For choice I, note that since f is an even function, f^{-1} doesn't exist. For choice II, f(h(-x)+g(-x)) = f(-[h(x)+g(x)]) = f([h(x)+g(x)]); so g(f([h(x)+g(x)])) = g(f([h(-x)+g(-x)])), and so choice II is even. For choice III, $g\left(h^{-1}\left(f(-x)\right)\right) = g(h^{-1}\left(f(x)\right)$ so choice III is odd; and for choice IV,
$g\left(f\left(g^{-1}\left(h(-x) + g(-x)\right)\right)\right) = g\left(f\left(g^{-1}(-[h(x) + g(x)])\right)\right) =$
$g\left(f\left(-g^{-1}([h(x) + g(x)])\right)\right) = g\left(f\left(g^{-1}([h(x) + g(x)])\right)\right)$ which is even, and $f\left(h^{-1}(x)\right) = f(-h^{-1}(x))$; the sum of even functions is even. Thus **II, III, IV.**
18. **A**—Consider the following diagram:



We know that side AB = 6, BC = 12. Because BC and BD are both radii of the circle centered at B, we also have BD = 12; since $\angle DAB = 90°$, we know that the length of DA is $6\sqrt{3}$. We can recognize this as a 30-60-90 right triangle, and so $\angle DBA = 60°$; hence, $\angle DBC = 30°$(since $\angle ABC = 90°$). Hence the area of triangle ADB is $18\sqrt{3}$ and the area of sector DBC is 12π. The area we want is the area of the rectangle minus this, or $72 - 18\sqrt{3} - 12\pi$; the overall answer is $\frac{72\cdot3^2}{18\cdot12} = 3$.
19. **A**—The areas of the annuli are $1^2, 2^2 - 1^2, 3^2 - 2^2, \ldots$; you'll notice that these are just 1, 3, 5, 7, … and so we need enough odd numbers to create a partition into three groups with each group having the same sum; if we have six numbers, we can pair 1 with 11, 3 with 9, and 5 with 7. Hence, n = 6.
20. **C**—the differences between terms are 6, 12, 18, 24; so the next two differences should be 30 and 36. Hence the two terms are 91 and 127; sum 218.
21. **C**—$(\sqrt{5} + \sqrt{6} + \sqrt{7})(-\sqrt{5} + \sqrt{6} + \sqrt{7}) = \left((\sqrt{7} + \sqrt{6})^2 - 5\right) = (8 + 2\sqrt{42})$; and $(\sqrt{5} - \sqrt{6} + \sqrt{7})(\sqrt{5} + \sqrt{6} - \sqrt{7}) = \left(5 - (\sqrt{7} - \sqrt{6})^2\right) = (-8 + 2\sqrt{42})$. The product of these two is $168 - 64 = 104$.

22. **B**— $N^2 = 2^{14}3^8$; this has $(14 + 1)(8 + 1) =$ 135 factors; 134 of these aren't N. Half of these 134 will be less than N and half are greater than N; hence, N^2 has 67 factors less than N. N has (7+1)(4+1)=40 factors, 39 of which are less than N (and all factors of N are factors of N^2); thus, there are 67 – 39 = 28 factors of N^2 which are less than N but not factors of N.
23. **B**—We need the sum of the digits to be a multiple of 3, since 15 is a multiple of 3; and the last digit must be a zero, since 15 is a multiple of 5.Hence the smallest number is 4440; the greatest integer less than $\frac{4440}{50}$ is 88.
24. **B**—Using the binomial theorem, we have $\binom{1337}{2}a^2b^{1335} = \binom{1337}{3}a^3b^{1334}$; hence, $\binom{1337}{2}b = \binom{1337}{3}a \to b = \frac{1335}{3}a = 445a \to a = 1, b = 445$; $a + b = 446$.
25. **C**—This can be done using either similar triangles or coordinate geometry; coordinate geometry is straightforward if a tad bit tedious, so we'll focus on similar triangles. Consider the diagram below:



We want the length of side BD. If we draw the line segment CD, we know that BDC is a right angle since BC is a diameter of the circle; hence, the ratio of AD to AC is the same as the ratio of AC to AB. We know that AC = 1, AB = $\sqrt{5}$; hence, AD = $\frac{1}{\sqrt{5}}$ and so $DB = \sqrt{5} - \frac{1}{\sqrt{5}} = \frac{4}{\sqrt{5}}$.
26. **D**—$a_1 = \frac{1}{1-x}, a_2 = \frac{1}{1-\frac{1}{1-x}} = \frac{1}{-\frac{x}{1-x}} = \frac{x-1}{x}$; $a_3 = \frac{1}{1-\frac{x-1}{x}} = \frac{1}{\frac{1}{x}} = x$; hence, the sequence repeats with a period of 3. Thus, $a_{2013} = a_3 = 2013$.
27. **C**—Let's expand both sides separately: $\log_8(\log_2 x) = \frac{1}{3}\log_2 \log_2 x$ and $\log_2(\log_8 x) = \log_2\left(\frac{1}{3}\log_2 x\right)$. Hence, $\log_2 \log_2 x = \log_2\left(\frac{1}{3}\log_2 x\right)^3$. If we let $y = \log_2 x$ then we have $y = \left(\frac{1}{3}y\right)^3 \to 27y = y^3$; we either have $y = 0$ or $y = \pm 3\sqrt{3}$; only $y = 3\sqrt{3}$ is a feasible solution (since if $y = 0$, then $\log_2 y$ is undefined).
28. **E**—$\frac{x+y}{2} = \sqrt{xy} + 8 \to x + y = 2\sqrt{xy} + 16 \to x - 2\sqrt{xy} + y = \left(\sqrt{y} - \sqrt{x}\right)^2 = 16$. Hence, $\sqrt{y} - \sqrt{x} = 4$ (since $y > x$). Since $x > 0$, we have $y \geq 5^2$; and since $y < 1000$, we have $y \leq 31^2$. Thus, y could be $5^2, 6^2, \ldots 31^2$; this is 27 total values.
29. **A**—Let's rewrite: $1234567 \times 9999999 = 1234567 \times (100000000 - 1)$=12345670000000 – 1234567 = 12345660000000 + (10000000 – 1234567); clearly this will have no 9s in it.
30. **C**—Don't just cross-multiply and expand the whole thing! Make the substitution $y = x^2 - 7x - 6$ so that the equation becomes $\frac{1}{y} + \frac{1}{y-8} - \frac{2}{y-12} = 0$. Then, cross-multiply to get a common denominator: $\frac{(y-8)(y-12)+y(y-12)-2y(y-8)}{y(y-8)(y-12)}$=0. We can clear the denominator and simplify the numerator to reduce this to $-16y + 96$ =0; hence, y = 6. Thus, $x^2 - 7x - 6 = 6 \to x^2 - 7x - 12 = 0$; the product of the roots is -12.