1. **D** $r^2 = x^2 + y^2 = a^2\theta^2(\cos^2(\theta) + \sin^2(\theta) = a^2\theta^2) = 36\theta^2$. To integrate in polar coordinates, we compute $\int_{2\pi}^{3\pi} \frac{r^2}{2} d\theta = \int_{2\pi}^{3\pi} 18\theta^2 d\theta = 114\pi^3$.

2. **C** Let $x = \frac{1}{\sqrt{2}}\sin(u)$. Then $\int_0^{\frac{1}{\sqrt{2}}} \frac{1}{\sqrt{1-2x^2}}dx = \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{1-\sin^2(u)}}\frac{1}{\sqrt{2}}\cos(u)du = \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{2}}du = \frac{\pi\sqrt{2}}{4}$

3. **A** $1 - 2 + \frac{2^2}{2} - \frac{2^3}{6} + \frac{2^4}{24} = \frac{1}{3}$.

4. **B** Let's figure out the formula for $\prod_{k=2}^{n} \frac{k^2}{k^2-1}$. Write $k^2 - 1$ as $(k-1)(k+1)$; then we have $\prod_{k=2}^{n} \frac{k^2}{k^2-1} = \frac{2\cdot 2}{1\cdot 3}\cdot\frac{3\cdot 3}{2\cdot 4}\cdots\frac{(n-1)\cdot(n-1)}{(n-2)\cdot n}\cdot\frac{n\cdot n}{(n-1)\cdot(n+1)}$. Notice that everything cancels except $\frac{2}{1}$ and $\frac{n}{n+1}$; $\lim_{n\to\infty}\frac{2n}{n+1} = 2$.

5. **D** $\int_{-\infty}^{\infty} 2^{-|x|}dx = \int_{-\infty}^{0} 2^x dx + \int_0^{\infty} 2^{-x}dx = \left[\frac{2^x}{\ln(2)}\right]_{-\infty}^{0} + \left[\frac{-2^{-x}}{\ln(2)}\right]_0^{\infty} = \frac{2}{\ln(2)}$.

6. **A** $\frac{dx}{dt} = \frac{e^{\sqrt{t}}}{2\sqrt{t}}; \frac{dy}{dt} = 3 - \frac{2}{t}$. At $t = 1$, then $x = e, y = 3$, $\frac{dx}{dt} = \frac{e}{2}$, $\frac{dy}{dt} = 1$. Hence, the slope of the tangent line is $\frac{2}{e}$; plugging in the point $(e, 3)$ we get $y = \frac{2}{e}x + 1$.

7. **D** $\frac{dx}{dt} = 6t$; $\frac{dy}{dt} = 6t^2$; $\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = 6t\sqrt{1+t^2}$. Hence, our arc length is $\int_0^1 6t\sqrt{1+t^2}dt$. Make the substitution $u = 1 + t^2$; then our integral becomes $\int_1^2 3\sqrt{u}du = \left[2u^{\frac{3}{2}}\right]_1^2 = 4\sqrt{2} - 2$.

8. **B** $\int_0^{\infty} (t+3)e^{-2t}dt = \int_0^{\infty} te^{-2t}dt + \int_0^{\infty} 3e^{-2t}dt$. Use integration by parts on the first integral; we obtain $\left[\frac{-2t-1}{4}e^{-2t} - \frac{3}{2}e^{-2t}\right]_0^{\infty} = \frac{7}{4}$.

9. **E** $\int_0^{\infty} e^t e^{-xt}dt = \int_0^{\infty} e^{(1-x)t}dt \to L(x) = \left[\frac{e^{(1-x)t}}{1-x}\right]_{t=0}^{\infty} = \frac{1}{x-1}$ (since $x > 1$, $e^{(1-x)t} \to 0$) which has range $(0, \infty)$.

10. **A** Let $3u = x^3 \to du = x^2dx$. Then we can rewrite this integral as $\int_{-\infty}^{\infty} \frac{du}{9u^2+9} = \frac{1}{9}\int_{-\infty}^{\infty}\frac{du}{u^2+1} = \frac{1}{9}\left[\arctan(u)\right]_{-\infty}^{\infty} = \frac{\pi}{9}$.

11. **E** III is true. I is false; suppose $f(x) = g(x) = \frac{1}{x}$. II is false: suppose $a_n = 2\left(\frac{1}{2}\right)^n$ and $b_n = (-1)^n$. Then every partial sum of a_n is greater than 2 and every partial sum of b_n is 1 if n is even or 0 if n is odd; but the sum $\sum_{n=0}^{\infty} b_n$ does not converge.

12. **A** Write the integrand as one fraction. If the degree of the top polynomial is not at least one less than the degree of the bottom polynomial, the integral diverges by comparison to $\int_1^{\infty}\frac{1}{x}dx$. Hence, we see that $C = 3$. Evaluating the integral, we get

$$\lim_{b\to\infty}\left[\frac{1}{2}\ln(x^2+1) - \ln(3x+1)\right]_0^b = \lim_{b\to\infty}\left[\frac{1}{2}\ln\frac{\sqrt{x^2+1}}{3x+1}\right]_0^b = \ln\frac{1}{3} - \ln 1 = -\ln(3).$$

13. **B** The region is a circle of diameter 3, so we integrate from 0 to π: $A = \int_0^{\pi}\frac{1}{2}(3\cos(\theta))^2d\theta = \frac{9}{2}\int_0^{\pi}\cos^2(\theta)d\theta = \frac{9\pi}{4}$.

14. **E** Let s_k be the k^{th} partial sum. Then $s_k = \ln(\frac{1}{k+1})$; hence, $\lim_{k\to\infty} s_k = -\infty$ and the sum is divergent.

15. **C** Notice that Q is the left-hand rectangle approximation and R is the right-hand approximation. Since $f(x)$ is decreasing, Q overestimates the integral and R underestimates; thus $R < P < Q$ (the inequality is strict because f is continuous).

16. **D** Series I and II have bounded, monotonically increasing partial sums (I is bounded by $\sum_{k=1}^{\infty} \frac{1}{2^n}$ and II is bounded by $\sum_{n=1}^{\infty} \frac{1}{n^2}$) and III converges by the Integral Test. Hence all converge.

17. **B** $\int_1^2 \frac{3}{x^2+3x}dx = \int_1^2 \left(\frac{1}{x} - \frac{1}{x+3}\right) dx = \left[\ln \frac{x}{x+3}\right]_1^2 = \ln(2/5) - \ln(1/4) = \ln \frac{8}{5}$

18. **C** The area of the region in question is $\int_0^1 (\sqrt{x} - x^2)dx = \frac{1}{3}$. The x-coordinate is thus $\frac{1}{\frac{1}{3}} \int_0^1 x(\sqrt{x} - x^2)dx = 3(\frac{2}{5} - \frac{1}{4}) = \frac{9}{20}$.

19. **C** The centroid of the triangle formed is the average of the coordinates, or (4, 2). The triangle has base 6 and height 3 $\Rightarrow$ area 9. We use the Theorem of Pappus: $V = 2\pi r A = 18\pi r$. Since (4, 2) is two units away from (4, 4), $r = 2$. Hence $V = 36\pi$.

20. **C** $\frac{dy}{dt} = 2t$; $\frac{dx}{dt} = 2\cos(t) \rightarrow \frac{dy}{dx} = \frac{t}{\cos(t)}$; $\frac{d^2y}{dx^2} = \frac{\left(\frac{\cos(t)+t\sin(t)}{\cos^2(t)}\right)}{2\cos(t)}$. Plugging in π, we get $\frac{1}{2}$.

21. **A** Let $u = \ln(t)$. Then $\int_{e^\pi}^{e^{2\pi}} \frac{\sin(\ln(t))}{t} dt = \int_\pi^{2\pi} \sin(u)du = -\cos(2\pi) + \cos(\pi) = -2$.

22. **A** $A = \frac{\pi a(6-a)}{4}$; $0 \le a \le 6$, so average area is $\frac{1}{6-0} \int_0^6 \frac{\pi a(6-a)}{4} da = \frac{\pi}{24} \int_0^6 (6a - a^2)da = \frac{3\pi}{2}$.

23. **B** $\lim_{n\to\infty} \sum_{k=1}^{n} \sqrt[3]{\frac{k^4}{n^7}} = \lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{n}\left(\frac{k}{n}\right)^{\frac{4}{3}} = \int_0^1 x^{\frac{4}{3}} dx = \frac{3}{7}$.

24. **A** The first three nonzero terms of the Maclaurin series are $1 - x^2/2 + x^4/24$. Plugging in $x = 2$ gives $\cos(x) = 1 - 2 + \frac{16}{24} = -\frac{1}{3}$. Hence $\sec(x) = -3 \rightarrow \tan^2(x) = 8 \rightarrow \tan(x) = -2\sqrt{2}$ (since $\frac{\pi}{2} < 2 < \pi$, this is a second-quadrant angle so tangent is negative).

25. **C** We use the Ratio Test:

$$\lim_{n\to\infty}\left|\frac{(2x-6)^{n+1}}{\sqrt{(n+1)^2+3}} \cdot \frac{\sqrt{n^2+3}}{(2x-6)^n}\right| = \lim_{n\to\infty}\left|(2x-6)\sqrt{\frac{n^2+3}{(n+1)^2+3}}\right| = |2x-6| \lim_{n\to\infty}\sqrt{\frac{n^2+3}{(n+1)^2+3}} = |2x-6|.$$

We need $|2x-6| < 1$, so $5 < 2x < 7 \rightarrow 2.5 < x < 3.5$; radius of convergence $\frac{1}{2}$.

26. **B** Plug each choice in, applying the integration by parts formula. Only choice (b) works.

27. **D** $[f(x)g(x)]' = f(x)g'(x) + g(x)f'(x) \rightarrow [f(x)g(x)]' - g(x)f'(x) = f(x)g'(x)$. Integrate both sides to obtain the integration by parts formula.

28. **D**

$$L = \int_0^{\frac{\pi}{4}} \sqrt{1+(f'(x))^2}dx = \int_0^{\frac{\pi}{4}} \sqrt{1+(-\tan(x))^2}dx = \int_0^{\frac{\pi}{4}} \sec(x)dx = \ln(\sqrt{2}+1) \rightarrow e^L = \sqrt{2}+1.$$

29. **B** $\mathbb{E}(X) = \int_0^\infty xce^{-cx}dx = \frac{1}{c}\int_0^\infty ue^{-u}du = \frac{1}{c}$ (use integration by parts on $\int_0^\infty ue^{-u}du$).

30. **D** From question 29 we know that $\mathbb{E}(X) = \frac{1}{c}$. Hence we compute $\mathbb{E}(X^2) = \int_0^\infty x^2ce^{-cx}dx = \frac{1}{c^2}\int_0^\infty u^2e^{-u}du$. Apply integration by parts on $\int u^2e^{-u}du$ twice to obtain $\int_0^\infty u^2e^{-u}du = 2$. Hence the variance is $2(\frac{1}{c^2}) - \left(\frac{1}{c}\right)^2 = \frac{1}{c^2}$.