Mu Ciphering

Problem	Question	Answer	Solution
0 – VE	What is the slope of the line $3x+4y=5$? Express your answer as a common fraction.	$-3/4$	Rewriting the equation of the line in slope-intercept form: $y=5/4-3/4\,x$. Hence, we realize that the slope is $-3/4$.
1 – CE	For what value of $x\in[-1,3]$ does $f'(x)$ equal the slope of the secant line connecting the points $(-1, f(-1))$ and $(3, f(3))$ for $f(x)=x^2-5x+7$?	$x=1$	By the Mean Value Theorem, we have that the value of $x=c$ that yields the slope of the specified secant line is: $f'(c)=\frac{f(3)-f(-1)}{(3)-(-1)}$ $2c-5=-3$ $\therefore c=1$ And hence, we have the solution $x=1$.
2 – NCE	What is the product of the positive integral factors of 16?	1024	Notice that the positive factors of 16 are just the powers of 2 from 2^0 to 2^4. Taking the product of these factors yields: $2^0*2^1*2^2*2^3*2^4=2^{10}=1024$
3 – CE	Evaluate the following limit: $\lim_{x\to 0}\frac{1-\sec^2}{x^2}$.	$\lim_{x\to 0}\frac{1-\sec^2}{x^2}=-1$	Apply L'Hospital's Rule twice.
4 – NCE	Evaluate: $x=\sqrt{10+3\sqrt{10+3\sqrt{\cdots}}}$.	$x=5$	If we square both sides, we obtain the equality: $x^2=10+3\sqrt{10+3\sqrt{\cdots}}$ $=10+x$ Hence, applying the quadratic formula, we obtain that $x=5$ or

			$x=-2$. Since $x>0$, we must have that $x=5$.
5 – NCM	Find the period of the graph of $y = 5\sin\left(\frac{x}{4}\right)\cos\left(\frac{x}{5}\right) + 12\cos(4x)\sin(5x)$.	40pi	The first term has a period of 40pi. The second term has a period of 2pi. The first period is a multiple of the second, so the period of the sum is just 40pi.
6 – NCM	What is the ratio between the areas of the inscribed hexagon and circumscribed hexagon of a circle?	$3/4$	Notice that the ratio between the linear dimensions of the inscribed hexagon and the circumscribed hexagon is $\sqrt{3}/2$. Since the area of a regular polygon scales quadratically compared to its linear size, the ratio between the areas is $3/4$.
7 – NCM	Out of the top 10 Mu Ciphering last year, 4 were female. What is the probability that 3 of them were in the top 5? Express your answer as a proper fraction.	$5/21$	The probability of obtaining 3 females in the top 5 placers is the number of ways three females can be in the top 5 divided by the total number of arrangements of the top 10 placers. Hence the probability is $P = \frac{\binom{6}{2}\binom{4}{3}}{\binom{10}{5}} = \frac{15\times 4}{252} = \frac{5}{21}$.
8 – CM	If the rate at which the sides of a cube are growing is 3 cm/sec, then how fast is the volume changing the instant when the surface area of the cube reaches 54 cm^2?	81 [$units^3$]	The volume of a cube is $V = x^3$, hence the rate at which the volume changes with respect to the rate of change for one of its side lengths is $\frac{dV}{dt} = 3x^2\frac{dx}{dt}$. So when the surface area is 54 cm squared, the side length of the cube is 3 cm. Therefore substituting in this value for x alongside the rate of growth for the side length of the cube, we obtain that the volume is changing at a rate of 81 cm cubed.

9 – CH	What is the surface area of the paraboloid created by rotating the graph of $y=\frac{1}{2}x^2$, defined on the interval $x\in[-\sqrt{3},\sqrt{3}]$, about the y-axis?	$\frac{14\pi}{3}$	By rotation of the graph $y=\frac{1}{2}x^2$ about the y-axis, we are motivated to consider horizontal slices of the paraboloid. Each slice is a thin ring, with surface area $2\pi x\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = 2\pi x\sqrt{1+x^2}\,dx$. Hence as we integrate over all of these ring slices of the paraboloid's surface, we find that the surface area is $A=\int dA_{ring}$ $=\int_0^{\sqrt{3}} 2\pi x\sqrt{1+x^2}\,dx$ $=2\pi\int_0^2 u^2\,du$ $=\frac{14\pi}{3}$ Where in the third equality, we pulled out all constants and made the u-substitution: $u=\sqrt{1+x^2}$.
12 – CH	What is the value of the integral: $I=\int_{-\infty}^{\infty} e^{-\lvert x\rvert}dx$?	$I=2$	Notice that we can rewrite the integral as $I=\int_{-\infty}^{0} e^{-x}dx+\int_{0}^{\infty} e^{x}dx$. Evaluating these two improper integrals, we realize that both integrals evaluate to 1 and hence their sum is 2.
11 – C	What is the average value of $f(x)=\cos^2 x$ on the interval $\left[0,3\pi\right]$?	$\frac{1}{2}$	The average value of a function over an interval is the integral of the function over given interval divided by the width of the interval. Hence,

			$AV = \frac{\int_0^{3\pi} \cos^2 x dx}{3\pi}$ $= \frac{\int_0^{3\pi} (1+\cos 2x) dx}{6\pi}$ $= \frac{1}{6\pi}\left(x + \frac{\sin 2x}{2}\right)\Big|_0^{3\pi}$ $= \frac{3\pi}{6\pi} = \frac{1}{2}$
10 – C	Evaluate: $\frac{d}{dx}\{f(f(f(1)))\}$ for $f(x) = x^2 + 5$.	1968	An easy way of quickly evaluating this expression is by applying the chain rule for differentiation: $\frac{d}{dx}\{f(f(f(1)))\} = \frac{df}{dx}(f(f(1)))\cdot\frac{df}{dx}(f(1))\cdot\frac{df}{dx}(1)$ $= 2((x^2+5)+5)\Big|_{x=1}\cdot 2(x^2+5)\Big|_{x=1}\cdot 2x\Big|_{x=1}$ $= 82\cdot 12\cdot 2$ $= 1968$
13 – C	The function $y(x)$ is the solution to the Initial Value Problem: $\frac{dy}{dx} = y(x), y(0) = 2.$ What is the value of $y(\ln 2)$?	4	Separating variables, we have that $\frac{dy}{y} = dx$. Integrating from $x' = 0$ and $x' = x$, we have that $\ln y = x + c$ or $y(x) = Ce^x$. Applying our initial condition, we realize that $C = 2$. Hence, our solution is $y(x) = 2e^x$. Therefore when the solution is evaluated, we find $y(\ln 2) = 4$.