MU ALPHA THETA

NATIONAL CONVENTION
SAN DIEGO 2013

Individual
Mu
Test #211

1. Write your 6-digit ID# in the I.D. NUMBER grid, left-justified, and bubble. Check that each column has only one number darkened.
2. In the EXAM NO. grid, write the 3-digit Test # on this test cover and bubble.
3. In the Name blank, print your name; in the Subject blank, print the name of the test; in the Date blank, print your school name (no abbreviations).
4. Scoring for this test is 5 times the number correct + the number omitted.
5. You may not sit adjacent to anyone from your school.
6. TURN OFF ALL CELL PHONES OR OTHER PORTABLE ELECTRONIC DEVICES NOW.
7. No calculators may be used on this test.
8. Any inappropriate behavior or any form of cheating will lead to a ban of the student and/or school from future national conventions, disqualification of the student and/or school from this convention, at the discretion of the Mu Alpha Theta Governing Council.
9. If a student believes a test item is defective, select “E) NOTA” and file a Dispute Form explaining why.
10. If a problem has multiple correct answers, any of those answers will be counted as correct. Do not select “E) NOTA” in that instance.
11. Unless a question asks for an approximation or a rounded answer, give the exact answer.

Note: For all questions, answer "(E) NOTA" means none of the above answers is correct.

1. This value of $f'(c)$ is guaranteed by the Mean Value Theorem for some c in the interval $x \in (1,4)$ when $f(x) = \sqrt{x}$.

(A) 1/3 (B) 9/4 (C) 1/2 (D) $\sqrt{2}/4$ (E) NOTA

2. Given that $f(x) = \log_3 x$, evaluate $f'(2)$.

(A) $\frac{e}{3 \ln 2}$ (B) $\log_2 3$ (C) $1/\ln 9$ (D) $\frac{2}{3}$ (E) NOTA

3. Evaluate $h'(x)$ where $h(x) = \frac{x^3+3x^2-8x-9}{e^x}$.

(A) $\frac{6x^2-4x+9}{xe^x}$ (B) $\frac{3x^2+6x-8}{e^x}$ (C) $\frac{-x^3+14x+1}{e^x}$ (D) $\frac{3(x-4)^2+7}{e^x}$ (E) NOTA

4. Evaluate: $\int (2t-3)^4 dt$

(A) $8(2t-3)+C$ (B) $\frac{(2t-3)^5}{10}+C$ (C) $\frac{8t(2t-3)^3}{5}+C$ (D) $\frac{8t(2t-3)^5}{5}+C$ (E) NOTA

5. Calculate the second-order Taylor polynomial of $f(x) = \sqrt{x}$ at $x = 1$ and use this polynomial to estimate $\sqrt{1.3}$ to the nearest thousandth.

(A) 1.137 (B) 1.139 (C) 1.140 (D) 1.150 (E) NOTA

6. This theorem states that if $f(x)$ is continuous on $x \in [a,b]$ and differentiable on $x \in (a,b)$ and if $f(a) = f(b)$, then there is some $c \in (a,b)$ such that $f'(c) = 0$.

(A) Extreme Value Theorem (B) Fundamental Theorem of Calculus

(C) Squeeze Theorem (D) Rolle's Theorem (E) NOTA

7. The *derivative* of a twice continuously differentiable function f is defined by $f'(x) = x^2(x+2)(x-3)^3 e^{x-1}$. The set of x-values at which f is a relative minimum is:

(A) $\{0, 3\}$ (B) $\{-2, 0, 3\}$ (C) $\{3\}$ (D) $\{0, 1\}$ (E) NOTA

8. Find the limit of the following sequence as k approaches ∞ : $\frac{8}{5}, \frac{5}{9}, \frac{2}{13}, \ldots, \frac{-3k+11}{4k+1}, \ldots$

(A) $3/4$ (B) $-3/4$ (C) 0 (D) $-1/4$ (E) NOTA

9. Determine the area enclosed by the polar graph $r = \sin\theta + \cos\theta$.

(A) $\pi/2$ (B) π (C) $2\pi/3$ (D) $2\pi\sqrt{3}$ (E) NOTA

10. Find the volume of the solid created by rotating about the x-axis the region between the lines $y = x^2 - 8x + 17$, $x = 3$, $x = 6$, and the x-axis.

(A) 12π (B) $\frac{45\pi}{4}$ (C) $\frac{56\pi}{5}$ (D) $\frac{78\pi}{5}$ (E) NOTA

11. Evaluate: $\int_{-1/2}^{1} \frac{1}{\sqrt{1-x^2}}\,dx$

(A) $\frac{2\sqrt{3}}{3}$ (B) $\frac{2\pi}{3}$ (C) $\frac{\sqrt{6}+\sqrt{2}}{4}$ (D) $\frac{\pi\sqrt{2}}{2}$ (E) NOTA

12. Write an equation of the line tangent to the graph of $y = f(g(x))$ at $x = 2$ if $f(x) = 2x^3 - 5x^2 + 8$ and $g(x) = -x^3 + 3x^2 - 6x + 9$.

(A) $4x - y = -1$ (B) $16x + y = 33$ (C) $24x - y = 43$ (D) $6x + y = 10$ (E) NOTA

13. Calculate: $\int_0^{\pi/3} 2\sec^2\left(\frac{\theta}{2}\right)\tan\left(\frac{\theta}{2}\right)d\theta$

(A) $\frac{2}{3}$ (B) $\frac{2\sqrt{3}}{3}$ (C) $\frac{3\sqrt{6}}{2}$ (D) 6 (E) NOTA

14. Evaluate: $\int \frac{1+x}{1+x^2}\,dx$

(A) $\ln|x-1| + C$ (B) $\frac{1}{2}\ln(x^2+1) + C$ (C) $x + \tan^{-1}x + C$ (D) $x + \ln(x^2+1) + C$ (E) NOTA

15. If a spherical snowball melts so that its surface area decreases at a rate of 2 in^2/min, find the rate at which the diameter decreases when the diameter is 5 inches. Assume that the snowball maintains a spherical shape as it melts.

(A) $\frac{\pi}{2}$ in/min (B) $\frac{3}{10\pi}$ in/min (C) $\frac{2\pi}{25}$ in/min (D) $\frac{1}{5\pi}$ in/min (E) NOTA

16. What is the coefficient of the x^3y^4 term in the expansion of $(2x - y + 3)^9$?

(A) 38,420 (B) 64,560 (C) 76,840 (D) 90,720 (E) NOTA

17. Express the following difference in base 20: $133.3_4 - 44.1_5$

(A) 8.2_{20} (B) $7.B_{20}$ (C) $5.D_{20}$ (D) $A.8_{20}$ (E) NOTA

18. What is the sum of the positive integral factors of 2,013?

(A) 2,976 (B) 5,989 (C) 4,284 (D) 3,396 (E) NOTA

19. Suppose the line $y = 4x - 5$ is tangent to the curve $y = f(x)$ when $x = 3$. If Newton's method is used to locate a root of the equation $f(x) = 0$ and the initial approximation is $x_1 = 3$, then find the second approximation x_2.

(A) 5/8 (B) $\frac{1+\sqrt{5}}{2}$ (C) 5/4 (D) 7/8 (E) NOTA

20. I am thinking of a natural number n. Given that following statements are true, find n. (Note that here, A and E are acting as digits.)

$14AE_n < 13000_{10}$
n is not a multiple of 4.
n has 3 proper positive integral factors.
The thousands digit of $n!$ is non-zero.

(A) 14 (B) 15 (C) 18 (D) 21 (E) NOTA

21. Solve for y in terms of x: $x = 2\tan\theta$ and $y = 4\cos^2\theta$ on the interval $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$.

(A) $\frac{\cos^2(\tan^{-1} x)}{2}$ (B) $2(x^2+2)$ (C) $\frac{16}{x^2+4}$ (D) $2x^3-6x+4$ (E) NOTA

22. Aaron lists the positive integers from 1 to 100, inclusive. Brian comes by and erases any prime number. Chloe then comes and erases any remaining multiple of 3. Denise follows suit and erases any remaining multiple of 5, and finally Ethan erases remaining multiples of 7. In the end, how many numbers remain?

(A) 15 (B) 23 (C) 26 (D) 27 (E) NOTA

23. In Linear Algebra, for a set U of vectors, this is defined as the set of all linear combinations of the vectors in U.

(A) Null space (B) Kernel (C) Span (D) Rank (E) NOTA

24. Let **A** be an $m \times n$ matrix and k an integer with $0 < k \leq m$, and $k \leq n$. A $k \times k$ *minor* of **A** is the determinant of a $k \times k$ matrix obtained from **A** by deleting $m - k$ rows and $n - k$ columns. How many minors, allowing for non-uniqueness, are in a 3x4 matrix?

(A) 22 (B) 34 (C) 49 (D) 56 (E) NOTA

25. The sum of the squares of the first and fourth terms of an arithmetic sequence is 1685. The sum of the squares of the second and third terms of the same sequence is 241. What is the product of the first four terms of this sequence?

(A) $-10{,}920$ (B) 10,920 (C) 35,920 (D) 46,920 (E) NOTA

26. Find the centroid of the region bounded by $y = 2\sqrt{x}$, the x-axis, and the line $x = 6$.

(A) $\left(\frac{7}{2}, \frac{\sqrt{14}}{2}\right)$ (B) $\left(3\sqrt[3]{2}, \sqrt{6} - \frac{3}{4}\right)$ (C) $\left(2\sqrt{3}, \frac{4\sqrt{2}}{3}\right)$ (D) $\left(\frac{18}{5}, \frac{3\sqrt{6}}{4}\right)$ (E) NOTA

27. Let $\{X_n, n \geq 1\}$ be a sequence of continuous random numbers. Each X_n is uniformly distributed on the interval $[0,1]$, and all X_n's are independent. We say that X_n is a *record* if its value is larger than that of all preceding $n - 1$ values in the sequence (i.e., if $X_n > X_k$ for all positive integers $k < n$). Find the probability that X_n is a record, for $n \geq 1$.

(A) $\left(\frac{1}{2}\right)^{n-1}$ (B) $\frac{1}{n}$ (C) $\frac{1}{n!}$ (D) ne^{1-n} (E) NOTA

28. In a lottery, a random sequence L is generated. Suppose L contains four distinct integers between 1 and 10, inclusive. Find the probability that L includes exactly one pair of consecutive integers.

(A) $\frac{4}{7}$ (B) $\frac{7}{24}$ (C) $\frac{1}{2}$ (D) $\frac{18}{35}$ (E) NOTA

29. In a certain city the temperature (in °F) t hours after 9 A.M. is modeled by the function $T(t) = 50 + \frac{20\pi}{11}\sin\frac{\pi t}{12}$. Find the average temperature, in °F, during the period from 9 A.M. to 5 P.M. in a single day. Round your answer to the nearest hundredth.

(A) 52.87 (B) 52.96 (C) 54.09 (D) 54.17 (E) NOTA

30. Find the area contained inside the curve in the plane with equation $8x^2 + 12xy + 13y^2 = 884$.

(A) $26\pi\sqrt{17}$ (B) $17\pi\sqrt{3}$ (C) $\frac{68\pi\sqrt{221}}{3}$ (D) 102π (E) NOTA