ANSWERS

(1) D D C C C	**(6)** B C A A A	**(11)** B A B C B
(16) C A A B B	**(21)** D B C A D	**(26)** B A C A C

SOLUTIONS

1. We have

$$\det\begin{pmatrix} -2 & 2 & 3 \\ -1 & 1 & 3 \\ 2 & 0 & -1 \end{pmatrix} = (-2)(1)(-1)+(2)(3)(2)+(0)(-1)(3)$$
$$-(2)(1)(3)-(2)(-1)(-1)-(0)(3)(-2)$$
$$=6,\ \boxed{\text{D.}}$$

2. We have

$$\sum_{n=0}^{\infty}\begin{pmatrix} \frac{1}{2^n} & \frac{1}{3^n} \\ \frac{1}{4^n} & \frac{1}{5^n} \end{pmatrix} = \begin{pmatrix} \sum_{n=0}^{\infty}\frac{1}{2^n} & \sum_{n=0}^{\infty}\frac{1}{3^n} \\ \sum_{n=0}^{\infty}\frac{1}{4^n} & \sum_{n=0}^{\infty}\frac{1}{5^n} \end{pmatrix}$$
$$= \begin{pmatrix} \frac{1}{1-\frac{1}{2}} & \frac{1}{1-\frac{1}{3}} \\ \frac{1}{1-\frac{1}{4}} & \frac{1}{1-\frac{1}{5}} \end{pmatrix}$$
$$= \begin{pmatrix} 2 & \frac{3}{2} \\ \frac{4}{3} & \frac{5}{4} \end{pmatrix}.$$

Therefore our answer is $2+\frac{3}{2}+\frac{4}{3}+\frac{5}{4}=\frac{73}{12}$, $\boxed{\text{D.}}$

3. The span of the matrices is

$$\alpha\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}+\beta\begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}=\begin{pmatrix}\alpha & 0\\ 0 & \beta\end{pmatrix},$$

where α and β are real numbers. This is equivalent to $\boxed{\text{C.}}$

4. Note that $\det(AB)=\det A\cdot\det B$ for matrices A and B. Using this, we have

$$\begin{aligned}\det\left[\begin{pmatrix}1 & 1\\ 1 & t\end{pmatrix}\begin{pmatrix}1 & 2\\ 2 & t^2\end{pmatrix}\right] &= \det\begin{pmatrix}1 & 1\\ 1 & t\end{pmatrix}\cdot\det\begin{pmatrix}1 & 2\\ 2 & t^2\end{pmatrix}\\ &=(t-1)(t^2-4)\\ &=t^3-t^2-4t+4.\end{aligned}$$

To minimize this, we differentiate this last expression and set the result equal to 0:

$$\begin{aligned}\left(t^3-t^2-4t+4\right)' &= 3t^2-2t-4\\ &=0\\ &\Rightarrow t=\frac{2\pm\sqrt{52}}{6}=\frac{1}{3}\pm\frac{\sqrt{13}}{3}.\end{aligned}$$

After checking concavities we have $t=\frac{1}{3}+\frac{\sqrt{13}}{3}$, $\boxed{\text{C.}}$

5. Note that $1,\cos(mx)$, and $\sin(nx)$ are each piecewise orthogonal over $[-\pi,\pi]$ for $m,n\in\mathbb{R}$. Thus, nearly every term in the expansion of the product will integrate to 0. We are only left with

$$\int_{-\pi}^{\pi}1\cdot 1dx=2\pi,\ \boxed{\text{C.}}$$

6. We have

$$\begin{aligned}A^2 &= \int_0^1 (x+1)\left(\frac{1}{x^2+1}\right)dx \\ &= \int_0^1 \frac{x}{x^2+1} + \frac{1}{x^2+1}\,dx \\ &= \int_0^1 \frac{x}{x^2+1}\,dx + \int_0^1 \frac{1}{x^2+1}\,dx \\ &= \left(\frac{1}{2}\ln\left(x^2+1\right)\right)_0^1 + \left(\tan^{-1}x\right)_0^1 \\ &= \frac{1}{2}\ln 2 + \frac{\pi}{4},\ \boxed{\text{B.}}\end{aligned}$$

7. Note that the diagonal entries of A are the first n odd numbers. Thus our answer is just

$$\begin{aligned}\operatorname{tr}(A) &= 1+3+\cdots+(2n-1) \\ &= n^2,\ \boxed{\text{C.}}\end{aligned}$$

8. We have

$$\begin{aligned}a\left(x^2-1\right)+b\left(x^2+5x+6\right)+c\left(x+3\right) &= x^2(a+b)+x(5b+c)+(-a+6b+3c) \\ &= 2x^2+7x+8.\end{aligned}$$

Thus we have the system

$$\begin{aligned}a+b&=2\\ 5b+c&=7\\ -a+6b+3c&=8\end{aligned}$$

which has a solution of $(a,b,c)=\left(\frac{5}{8},\frac{11}{8},\frac{1}{8}\right)$. For a proper integral spanning, however, we must have integral a,b,c. Thus, we require that $n=8$, $\boxed{\text{A.}}$

9. The vectors will be orthogonal if their dot product is equal to $0.$ Each vector has 2^n possibilities, giving $\left(2^n\right)^2 = 4^n$ possible dot products. Consider each term of the vectors one by one. We have 3 possibilities for these two terms to multiply out to $0:$ either the term of the first vector is $0,$ the term of the second vector is $0,$ or both terms are $0.$ Since this condition is uniform among all the terms, there are 3^n possibilities where the dot product comes out to $0.$ This gives a probability of $\frac{3^n}{4^n} = \left(\frac{3}{4}\right)^n$, $\boxed{\text{A.}}$

10. The eigenvectors of A form the columns of $P.$ Since $Av = \lambda v$ for eigenvectors v and eigenvalues $\lambda.$ Thus, we have

$$\begin{pmatrix} -1 & 6 & 6 \\ -1 & 4 & 2 \\ 0 & 0 & 2 \end{pmatrix}\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix} \Rightarrow \lambda_1 = 2$$
$$\begin{pmatrix} -1 & 6 & 6 \\ -1 & 4 & 2 \\ 0 & 0 & 2 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ -2 \end{pmatrix} \Rightarrow \lambda_2 = 2$$
$$\begin{pmatrix} -1 & 6 & 6 \\ -1 & 4 & 2 \\ 0 & 0 & 2 \end{pmatrix}\begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} \Rightarrow \lambda_3 = 1$$

Since the eigenvalues form the entries in the diagonal of the diagonal matrix $D,$ we have that the trace is equal to $\text{tr}\left(A\right) = 2+2+1 = 5,$ $\boxed{\text{A.}}$

11. We can write $v_3 = -v_4 - v_5.$ Thus, v_1 is dependent on one parameter, while v_3 is dependent on two parameters, giving a total of three vectors in the basis. We can write the basis as $\left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ -1 \end{pmatrix} \right\}$, $\boxed{\text{B.}}$

12. After performing row operations, we find that

$$\text{rref}\left(\begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 4 & 3 & 6 \\ 2 & 0 & 1 & 0 \\ -1 & 2 & 1 & 3 \\ 1 & -2 & 1 & -1 \end{bmatrix}\right) = \begin{pmatrix} 1 & 0 & 0 & -0.5 \\ 0 & 1 & 0 & 0.75 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \boxed{\text{A.}}$$

13. The rank is the number of nonzero rows in the row-reduced echelon form, or 3, $\boxed{\text{B.}}$

14. The eigenvalues of A satisfy $\det(A-\lambda I)=0.$ Similarly, for $C,$ we have

$$\begin{aligned}\det(C-\lambda I) &= \det\left(B^{-1}AB-\lambda I\right) \\ &= \det\left(B^{-1}AB-\lambda IB^{-1}B\right) \\ &= \det\left(B^{-1}(AB-\lambda IB)\right) \\ &= \det\left(B^{-1}\right)\det(B)\det(A-\lambda I) \\ &= \det(A-\lambda I).\end{aligned}$$

Thus, A and C have the same eigenvalues. The sum of the eigenvalues of C is then 23, $\boxed{\text{C.}}$

15. Writing the vectors as rows of a matrix, we have $A=\begin{pmatrix} 1 & 2 & 1 & 3 \\ 3 & 6 & 3 & 9 \\ 1 & 3 & 5 & 4 \\ 2 & 3 & -2 & 5 \end{pmatrix}.$ The dimension of the basis will be equal to the rank of $A.$ Since $\text{rref}(A)=\begin{pmatrix} 1 & 0 & -7 & 1 \\ 0 & 1 & 4 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$ we have $\text{rank}(A)=\dim(B)=2,$ $\boxed{\text{B.}}$

16. The two vectors have a span of $\begin{pmatrix} a & b \\ a & b \end{pmatrix}$ for $a, b \in \mathbb{R}$. The only vector given that matches this is $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, $\boxed{\text{C.}}$

17. The null space is given by the set of vectors $v = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ such that $Av = 0$. Thus, we have

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

This forces a and b to be 0, while putting no restrictions on c. Thus a basis describing this space is $\left\{\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right\}$, $\boxed{\text{A.}}$

18. We must solve $\det(A - \lambda I) = 0 \Rightarrow \begin{vmatrix} 3-\lambda & -2 \\ 4 & -1-\lambda \end{vmatrix} \Rightarrow (3-\lambda)(-1-\lambda) + 8 = 0$. This gives the quadratic $\lambda^2 - 2\lambda - 3 + 8 = \lambda^2 - 2\lambda + 5 = 0$. Using the quadratic formula, we have

$$\lambda = \frac{2 \pm \sqrt{(-2)^2 - 4 \cdot 5}}{2} = 1 \pm 2i, \ \boxed{\text{A.}}$$

19. We wish to solve

$$\int_{-2}^{2} \left(x + c_1 x^2 + c_2 x^3\right) x\,dx = 0$$

$$\int_{-2}^{2} \left(x + c_1 x^2 + c_2 x^3\right) x^2\,dx = 0$$

Using the fact that f_1 and f_2 are orthogonal over this range, we can simplify the integrals to

$$\int_{-2}^{2} x^2 + c_2 x^4 dx = 0 \Rightarrow \left.\frac{x^3}{3}\right|_{-2}^{2} + c_2 \left.\frac{x^5}{5}\right|_{-2}^{2} = \frac{16}{3} + \frac{64}{5}c_2 = 0$$
$$\int_{-2}^{2} c_1 x^4 + c_2 x^5 dx = 0 \Rightarrow c_1 \left.\frac{x^5}{5}\right|_{-2}^{2} + 0 = 0$$

Solving these equations gives $(c_1, c_2) = \left(0, -\frac{5}{12}\right)$. Therefore our answer is $12c_2 = -5$, $\boxed{\text{B.}}$

20. Since we are just considering a 2×2 matrix, it's easiest just to do the summation out. First, consider $k = 0$. We only have one choice for A_k, namely $A_0 = A$. This means $P(A_0) = (a+b)(c+d) = \Sigma_0$. For $k = 1$, we have two choices for A_k, namely $\begin{pmatrix} a \\ c \end{pmatrix}$ and $\begin{pmatrix} b \\ d \end{pmatrix}$. This gives $P(A_1) = ac, bd \Rightarrow \Sigma_1 = ac + bd$. Thus, the permanent is

$$\begin{aligned}\text{perm}(A) &= \sum_{k=0}^{1} (-1)^k \Sigma_k \\ &= (-1)^0 (a+b)(c+d) + (-1)^1 (ac+bd) \\ &= ad + bc, \ \boxed{\text{B.}}\end{aligned}$$

21. This is just

$$\begin{aligned}L(g) &= g' + g \\ &= 2e^{2x}(2\cos x + 3\sin x) + e^{2x}(3\cos x - 2\sin x) \\ &\quad + e^{2x}(2\cos x + 3\sin x) \\ &= e^{2x}(9\cos x + 7\sin x), \ \boxed{\text{D.}}\end{aligned}$$

22. In the general case, we have

$$\begin{aligned} L(f) &= f' + f \\ &= 2e^{2x}(\alpha\cos x + \beta\sin x) + e^{2x}(-\alpha\sin x + \beta\cos x) \\ &\quad + e^{2x}(\alpha\cos x + \beta\sin x) \\ &= e^{2x}\left[(3\alpha+\beta)\cos x + (3\beta-\alpha)\sin x\right]. \end{aligned}$$

We can write f and $L(f)$ as vectors in terms of the given basis as $\begin{pmatrix}\alpha \\ \beta\end{pmatrix}$ and $\begin{pmatrix}3\alpha+\beta \\ 3\beta-\alpha\end{pmatrix} = \alpha\begin{pmatrix}3 \\ -1\end{pmatrix} + \beta\begin{pmatrix}1 \\ 3\end{pmatrix}$. Thus L can be represented as $L = \begin{pmatrix}3 & 1 \\ -1 & 3\end{pmatrix}$, B.

23. Note that $y' + y = L(y)$. Using the matrix we found in the previous problem, we can write this as $\begin{pmatrix}3 & 1 \\ -1 & 3\end{pmatrix} y = \begin{pmatrix}1 \\ 0\end{pmatrix}$. Letting $y = \begin{pmatrix}a \\ b\end{pmatrix}$ gives

$$\begin{cases} 3a + b = 1 \\ -a + 3b = 0 \end{cases} \Rightarrow (a,b) = \left(\frac{3}{10}, \frac{1}{10}\right).$$

Thus, a solution is $y = \frac{3}{10}\cos x + \frac{1}{10}\sin x$. Thus, our answer is $\dfrac{\frac{3}{10}}{\frac{1}{10}} = 3$, C.

24. We have to solve

$$M\begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_{n-1} \\ a_n \end{pmatrix} = \begin{pmatrix} a_1 \\ 2a_2 \\ \vdots \\ na_n \\ 0 \end{pmatrix}.$$

Clearly, M is an $n\times n$ matrix. Determining M can be done systematically. It is easy to see that the first row of M contains only 1 in the second entry. Continuing in this way, we find that $M=\begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 2 & \ddots & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \ddots & n \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix}$. We can write this as $M_{i,j}=\begin{cases} i, & \text{if } j=i+1 \\ 0, & \text{if } j\neq i+1 \end{cases}$, $\boxed{\text{A.}}$

25. We have to find reals c_1, c_2 such that

$$c_1\begin{pmatrix}1\\2\end{pmatrix}+c_2\begin{pmatrix}3\\5\end{pmatrix}=\begin{pmatrix}-2\\4\end{pmatrix}$$
$$\Rightarrow\begin{cases} c_1+3c_2=-2 \\ 2c_1+5c_2=4 \end{cases}$$

Solving this results in $(c_1,c_2)=(22,-8)$, which gives the vector $\begin{pmatrix}22\\-8\end{pmatrix}$, $\boxed{\text{D.}}$

26. Consider a vector in $\mathbb{R}^3$, $\begin{pmatrix}x\\y\\z\end{pmatrix}$. Applying T to this vector results in

$$\begin{aligned} T\begin{pmatrix}x\\y\\z\end{pmatrix} &= \begin{pmatrix}3 & 6 & -3\\ 1 & 2 & 1\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix} \\ &= \begin{pmatrix}3x+6y-3z\\ x+2y+z\end{pmatrix} \\ &= x\begin{pmatrix}3\\1\end{pmatrix}+y\begin{pmatrix}6\\2\end{pmatrix}+z\begin{pmatrix}-3\\1\end{pmatrix} \\ &= (x+2y)\begin{pmatrix}3\\1\end{pmatrix}+z\begin{pmatrix}-3\\1\end{pmatrix}. \end{aligned}$$

Thus, the set of vectors is $\left\{\begin{pmatrix}3\\1\end{pmatrix},\begin{pmatrix}-3\\1\end{pmatrix}\right\}$, $\boxed{\text{B.}}$

27. Note that $\|v\| = x^2 + y^2 + z^2 + 4.$ Furthermore note that $2(xy+yz+xz)+4(x+y+z) = 2(xy+yz+xz+2x+2y+2z),$ so we can write

$$2(xy+yz+xz+2x+2y+2z) = (x+y+z+2)^2 - (x^2+y^2+z^2+4),$$

where we have used the given equation. Thus we have

$$\begin{aligned} 4(x^2+y^2+z^2+4) &= (x+y+z+2)^2 \\ &\Rightarrow 4\|v\|^2 = (x+y+z+2)^2 \\ &\Rightarrow 2\|v\| = x+y+z+2. \end{aligned}$$

Consider the vector $w = (1,1,1,1).$ Then we can write the last equation as $\langle v, w\rangle = \|v\|\cdot\|w\|.$ This equation is the equality case for the Cauchy-Schwarz Inequality, and thus we know that v and w are collinear. This implies that $v = aw \Rightarrow (x,y,z,2) = (a,a,a,a) \Rightarrow a = 2.$ Thus, $x,y,z = 2,$ so we have $xyz = 2^3 = 8,$ $\boxed{\text{A.}}$

28. Consider the two vectors $v_1 = (a_1, a_2, \ldots, a_n)$ and $v_2 = (a_2, a_3, \ldots, a_n, a_1).$ Since we are given $a_1^2 + a_2^2 + \cdots + a_n^2 = 2,$ we know that both of these vectors have magnitude $\sqrt{2}.$ Thus, by the Cauchy-Schwarz Inequality, we have

$$\begin{aligned} &\langle v_1, v_2\rangle \le \|v_1\|\cdot\|v_2\| \\ &\Rightarrow a_1a_2 + a_2a_3 + \cdots + a_{n-1}a_n + a_na_1 \le \left(\sqrt{2}\right)^2 = 2, \ \boxed{\text{C.}} \end{aligned}$$

29. Note that $A^2 = \begin{pmatrix} 0 & 0 & 18 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ and $A^3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, which implies that $A^n = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ for $n \ge 3.$ Thus, $e^A = I + A + \frac{A^2}{2} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 3 & 4 \\ 0 & 0 & 6 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & \frac{18}{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 3 & 13 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{pmatrix},$ $\boxed{\text{A.}}$

30. We can write B_n as

$$B_n = I + A_n + \frac{1}{2}A_n^2 + \frac{1}{6}A_n^3 + \cdots$$
$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & -n \end{pmatrix} + \frac{1}{2}\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & -n \end{pmatrix}^2 + \frac{1}{6}\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & -n \end{pmatrix}^3 + \cdots$$

Note that $\begin{pmatrix} a_1 & 0 & 0 & 0 \\ 0 & a_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & a_n \end{pmatrix}^k = \begin{pmatrix} a_1^k & 0 & 0 & 0 \\ 0 & a_2^k & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & a_n^k \end{pmatrix}$, so we have

$$B_n = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & -n \end{pmatrix} + \frac{1}{2}\begin{pmatrix} (-1)^2 & 0 & 0 & 0 \\ 0 & (-2)^2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & (-n)^2 \end{pmatrix} + \cdots$$

This is similar to the Taylor Series expansion for e, so we have

$$\begin{pmatrix} 1+(-1)+\frac{1}{2}(-1)^2+\frac{1}{6}(-1)^3+\cdots & 0 & 0 & 0 \\ 0 & 1+(-2)+\frac{1}{2}(-2)^2+\frac{1}{6}(-2)^3+\cdots & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 1+(-n)+\frac{1}{2}(-n)^2+\frac{1}{6}(-n)^3+\cdots \end{pmatrix}$$
$$= \begin{pmatrix} \sum_{i=0}^{\infty}\frac{(-1)^i}{i!} & 0 & 0 & 0 \\ 0 & \sum_{i=0}^{\infty}\frac{(-2)^i}{i!} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \sum_{i=0}^{\infty}\frac{(-n)^i}{i!} \end{pmatrix}.$$

Of course, this is equal to $B_n = \begin{pmatrix} e^{-1} & 0 & 0 & 0 \\ 0 & e^{-2} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & e^{-n} \end{pmatrix}$. Thus, we have a trace of

$$\begin{aligned} \operatorname{tr}(B_n) &= e^{-1} + e^{-2} + \cdots + e^{-n} \\ \Rightarrow \lim_{n\to\infty}\left[\operatorname{tr}(B_n)\right] &= \sum_{i=1}^{\infty} e^{-i}. \end{aligned}$$

This is equal to $\dfrac{e^{-1}}{1-\frac{1}{e}} = \dfrac{1}{e-1}$, $\boxed{\text{C.}}$