Note: For all questions, answer "(E) NOTA" means none of the above answers is correct.

1. Find the common difference of the arithmetic sequence: 1, 4, 7, …

(A) 1 (B) 2 (C) 3 (D) 4 (E) NOTA

2. Let $i = \sqrt{-1}$. Consider the sequence $a_n = \cos\left(\frac{\pi n}{3}\right) + i\sin\left(\frac{\pi n}{3}\right)$ for integers $n \geq 1$. Find the value of a_{2013}.

(A) $-i$ (B) 1 (C) i (D) -1 (E) NOTA

3. Evaluate: $\sum_{n=0}^{\infty} \frac{(-1)^n 2^n}{(2n)!}$

(A) $\sin\sqrt{2}$ (B) $\cos\sqrt{2}$ (C) $\cos 2$ (D) $\sin 2$ (E) NOTA

4. In terms of x, find the 51st term of the following arithmetic sequence: $3x$, $6x + 1$, …

(A) $101x + 49$ (B) $156x + 51$ (C) $103x + 52$ (D) $153x + 50$ (E) NOTA

5. If the first term of an arithmetic sequence is 1 and the fifth term of the sequence is 60, what is the common difference of the sequence?

(A) $\sqrt{2\sqrt{15}}$ (B) 15 (C) 4 (D) $\frac{59}{4}$ (E) NOTA

6. Find the limit of the sequence $a_x = \frac{4x^2+2}{5x^2+3^x+9}$ as x approaches positive infinity.

(A) 1 (B) 0 (C) ∞ (D) $\frac{4}{5}$ (E) NOTA

7. Consider the geometric sequence $a_1, a_2, \ldots$, where $a_1 = \sin x$ and $a_2 = \sin^2 x$. If $\sum_{k=1}^{\infty} a_k = \frac{1}{3}$, find a possible value of x, where $x \in \left[0, \frac{\pi}{2}\right]$.

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$ (E) NOTA

8. What is the sum of the first 25 smallest positive perfect squares?

(A) 4550 (B) 8725 (C) 6150 (D) 5525 (E) NOTA

9. Find the radius of convergence of $f(x)=\sum_{n=0}^{\infty}\frac{(-1)^n 3^n (x+10)^{4n}}{(2n+1)}$.

(A) 0 (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) ∞ (E) NOTA

10. The sum of seven positive numbers is 21. Find the smallest possible value of the arithmetic mean of the squares of these numbers.

(A) 10 (B) 11 (C) 12 (D) 13 (E) NOTA

11. Find the sum of all possible values for the first term of a geometric sequence with second term $1+i$ and fifth term $2+2i$. Note that here, $i=\sqrt{-1}$.

(A) 0 (B) $\frac{2(1+i)}{3}$ (C) $\frac{\sqrt[3]{4}}{2}(1+i)$ (D) 2 (E) NOTA

12. Which of the following infinite series converges?

(A) $\sum_{n=2}^{\infty}\frac{2013}{n\ln(n^2)}$ (B) $\sum_{k=0}^{\infty}\frac{(-2)^k k^k}{3k+7}$

(C) $\sum_{i=10}^{\infty}\frac{5i-4}{2i+1}$ (D) $\sum_{m=1}^{\infty}\frac{3m^{19}+5}{m^{20}+6}$ (E) NOTA

13. The sum of the first twenty terms of an arithmetic sequence is 1090. The 20^{th} term is 102. Find the first term of the sequence.

(A) 7 (B) 19 (C) 20 (D) 211 (E) NOTA

14. Find the sum of all values of x such that $x-1$, $2x$, and $5x+3$ form a geometric sequence of ***positive*** real numbers.

(A) 3 (B) 2 (C) 1 (D) 0 (E) NOTA

15. Evaluate: $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{(k-1)^2}{n^3}$

(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) 3 (D) 2 (E) NOTA

16. The roots of $24x^3-14x^2+kx+3=0$ form a geometric sequence of real numbers. Find the value of $|k|$.

(A) 9 (B) 8 (C) 7 (D) 6 (E) NOTA

17. Consider the equation $x^3 + Ax^2 + Bx + C = 0$. If the three roots of the equation, when listed from smallest to largest, form an arithmetic sequence of distinct positive integers, which of the following is necessarily true?

(A) The values of A, B, and C are all even.

(B) The value of C is a multiple of 3.

(C) The value of B is a multiple of 3.

(D) The value of A is a multiple of 3.

(E) NOTA

18. Find the coefficient of the t^4-th term of $\int_0^t (3x-1)^{10}\,dx$ when expanded and like-terms combined.

(A) -810 (B) 15 (C) -135 (D) 32805 (E) NOTA

19. Evaluate: $\frac{2}{3} + \frac{5}{9} + \frac{8}{27} + \cdots + \frac{3n-1}{3^n} + \cdots$

(A) 2 (B) $\frac{7}{4}$ (C) $\frac{6}{5}$ (D) $\frac{13}{10}$ (E) NOTA

20. The sum of an infinite geometric series is $2013 = 3 \times 11 \times 61$. Each of the terms in this series is squared, resulting in a series with a sum of $66429 = 2013 \times 33$. Find the common ratio of the original series.

(A) $\frac{26}{27}$ (B) $\frac{28}{29}$ (C) $\frac{30}{31}$ (D) $\frac{32}{33}$ (E) NOTA

21. If $\sum_{n=1}^{\infty} a_n$ is a conditionally convergent series, what is the radius of convergence of $f(x) = \sum_{n=1}^{\infty} a_n x^n$?

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) ∞ (E) NOTA

22. Let C equal the sum of the first 2013^{2013} smallest positive perfect cubes. Which of the following is equal to the number of positive integral factors of C?

(A) 238 (B) 240 (C) 242 (D) 244 (E) NOTA

23. Evaluate: $\sum_{n=1}^{100}(5n+1) - \sum_{n=1}^{100} 5n + 1$

(A) 101 (B) 100 (C) 99 (D) 0 (E) NOTA

24. The Exponential Function can be extended to matrices! More specifically, given a matrix M, its *exponential* e^M can be defined as $e^M = \sum_{n=0}^{\infty} \frac{1}{n!} M^n$. If $M = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$, evaluate: e^M.

(A) $\begin{bmatrix} e & 1 \\ 0 & e-2 \end{bmatrix}$ (B) $\begin{bmatrix} e & 1 \\ e & e^2 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 0 \\ e-1 & e^2 \end{bmatrix}$ (D) Me^2 (E) NOTA

25. Which of the following can one compute in order to find the number of ways to obtain a sum of 500 when rolling 2013 fair, six-sided dice? Assume the dice are distinguishable from each other.

(A) The exponent of the term with coefficient 2013 in the expansion of $(\sum_{n=1}^{6} x^n)^{500}$.

(B) The coefficient of x^{2013} of the expansion of $(x + x^2 + x^3 + x^4 + x^5 + x^6)^{500}$.

(C) The exponent of the term with coefficient 500 in the expansion of $(\sum_{n=1}^{6} x^n)^{2013}$.

(D) The coefficient of x^{500} of the expansion of $(x + x^2 + x^3 + x^4 + x^5 + x^6)^{2013}$.

(E) NOTA

26. A sequence of complex numbers is defined for integers $n \geq 1$ by $a_n = a_{n-1}/\overline{a_{n-1}}$, where $\overline{a_{n-1}}$ is the conjugate of a_{n-1} and $i = \sqrt{-1}$. If $a_{2013} = 1$, how many possible complex-numbered values are there for a_1, given that $|a_1| = 1$?

(A) 1 (B) 2012 (C) 2^{2012} (D) 2 (E) NOTA

27. If $f(x) = 2x + 3 + 2013 \sum_{n=0}^{\infty} \frac{(-1)^n 2^n x^n}{n!}$, evaluate: $2f''(5) + 3f'(5) - 2f(5)$

(A) 0 (B) -20 (C) 13 (D) -2013 (E) NOTA

28. The inhabitants of a faraway planet communicate using a language consisting of an alphabet of only four letters: A, B, C, and D. A valid word in this language consist of an even amount of As and an even amount of Bs, and any number of Cs and Ds. Some examples of valid words are AABB, AABBBBC, and AABBDC. How many valid four-letter words are in this language?

(A) 64 (B) 68 (C) 72 (D) 76 (E) NOTA

29. The geometric sequence $x_1, x_2, x_3, \ldots, x_{10}$ is strictly increasing and has terms consisting of only integer powers of 2. If we know that

$$\sum_{n=1}^{10} \log_2(x_n) = 500$$

and

$$90 < \log_2\left(\sum_{n=1}^{10} x_n\right) < 100,$$

find the value of $\log_2(x_{20})$.

(A) 185 (B) 190 (C) 195 (D) 200 (E) NOTA

30. Given that, for integers $n \geq 0$, we have a sequence of definite integrals a_n defined by:

$$a_n = \int_{-1}^{1} (x^2 - 1)^n \, dx,$$

Evaluate: $\lim_{n\to\infty} (a_n / a_{n-1})$

(A) $\sqrt{2}$ (B) -1 (C) $\frac{\pi}{2}$ (D) 1 (E) NOTA