Theta Angle Chasing – Solution Guide

1. **B:** x = 20, and the three angles of the triangles are 50, 55, and 75.
2. **B:** Arc AC has length 18pi, which is half of the circumference (180 degrees), so angle B has measure equal to 180/2 = 90.
3. **C:** The three regular polygons that tessellate are the triangle (interior angle = 60), the square (90) and hexagon (120). 60+90+120 = 270.
4. **B:** The given two angles have measure 81 and 57, so the third angle has measure 42 (XLII).
5. **D:** All three terms describe AD. It bisects angle A, and it is the perpendicular bisector of side BC.
6. **C:** The measure of angle APD is the average of the two arcs, or 80 degrees, and angle APC is the supplement to this angle.
7. **A:** The measure of angle ABD is half of the length of arc AD.
8. **E:** Since both of these angles share arc BC, they have the same measure, and their difference is zero.
9. **B:** Angle F has the same angle as angle C, which is a base angle of an isosceles right triangle (45-45-90).
10. **A:** A = 40, B = 108, C = 90.
11. **A:** The entire circumference is 20*pi, so the angle has measure 360/20 = 18.
12. **B:** The four angles are 24, 48, 96, and 192.
13. **D:** The other name for it is an undecagon.
14. **C:** They are perpendicular.
15. **D:** Let the first line intersect AC at D and BC at E, and the second line intersect BC at F and AB at G. We are given that angle CDE is 40, and we know that angle C is 60 (equilateral triangle), so angle CED is 80. This means that angle BFG is 100, and we know that angle B is 60. Therefore angle BGF is 20, so the bigger angle at that intersection is 180 – 20 = 160.
16. **B:** Let r be the radius of the circle. We know that x = pi*r^2. The perimeter of the triangle is 3*sqrt(3)*r. When you solve the first equation for r and combine the two equations, you get the answer given.
17. **A:** Angle D is the same as the third (ungiven) angle of triangle ABC, so it has measure 30. The percent of the circle's area in the triangle is the same as the percent of 360 degrees that the angle is. 30/360 = 0.833
18. **B:** The angle at vertex B in triangle ABC has measure 50, and in triangle BCD has measure 100. The ratio of percents is just the ratio of these angles, so 50/100 = .5
19. **B:** To represent the bounce off of wall AB (which we assume to be a perfect reflection), we can draw the reflection of the triangle over this wall. Now, we want to aim for the reflection of pocket C. If the length of a side of the table is x, then the ball needs to go up sqrt(3)/2*x and over x, so the tangent of the angle at which it must be hit is x/(sqrt(3)/2*x), or 2/sqrt(3).
20. **C:** CD is the perpendicular bisector to XY, so the angle between them will be a right angle.
21. **E:** AB, AC, and BC are all the same length, sqrt(2), so the triangle formed by the three points is equilateral, and all angles are 60 degrees.

22. **E:** In spherical geometry, the sum of the angles of a triangle must be strictly greater than 180.
23. **D:** Because angle B is a right angle and the trapezoid is a right trapezoid, angle C must also be a right angle. Therefore, diagonal AC creates an isosceles right triangle ABC, which means that side BC must be the same length as side AB, or 2. To find the length of BD, we see that it is the hypotenuse of a right triangle with legs of length 2 (side BC) and 3 (side CD). Therefore, diagonal BD has a length of sqrt(13).
24. **C:** I approached this problem by using the fact that the opposite angles in a cyclic quadrilateral add up to 180 degrees. Because ABCD forms a cyclic quadrilateral, and it can be shown that 1 and 3 are the only choices of the 6 possible similar-triangle statements that actually work out.
25. **B:** Triangle DEF is an isosceles triangle. The big arc between D and F on the circle is 360 – 84 = 276 degrees, so angle DEF is half of that (138). The remaining two angles of triangle DEF are the same, and add up to 180 – 138 = 42, so the measure of one of the angles is 21.
26. **C:** The interior angle of a regular decagon is 144 degrees. If we draw in the seven potential diagonals through point P, this divides the angle at P into 8 equal angles of measure 18. The minimum possible angle between two diagonals is this small angle (18), and the maximum possible angle is if we choose two diagonals farthest apart – this will contain 6*18 degrees, or 108. 108 – 18 = 90.
27. **C:** For the three points to not form a triangle, they have to be collinear. If the slope between A and C is the same as the slope from B to C, then these are sufficient and necessary conditions for ABC to be collinear. The slope between A and C is given by (-4 – (3 – t)) / (4 – 2t), and the slope between B and C is given by (-4 – (1 – t)) / (4 – (t + 2)). Setting these two fractions equal to each other and solving using cross-multiplication gives us t^2 – 5t + 6 = 0, which has two solutions: t = 2 and t = 3.
28. **C:** A and B are true because they use parallel lines, which give equivalent angles for the triangles. D is true, and this can be shown by using a variation of Pappus' theorem involving parallel lines instead of intersecting lines.
29. **E:** All four of the given angles are the same, because this is the construction for a Brocard point.
30. **D:** Let the measure of angle AGF be alpha, and the measure of angle BAG be beta. Triangle AFG is an isosceles triangle with base angles of measure alpha, so the vertex angle is (180 – 2 * alpha). This vertex angle, plus two angles equal to beta in measure, is equal to the entire interior angle of the pentagon, so 180 – 2 * alpha + 2 * beta = 108. This simplifies to alpha – beta = 36.