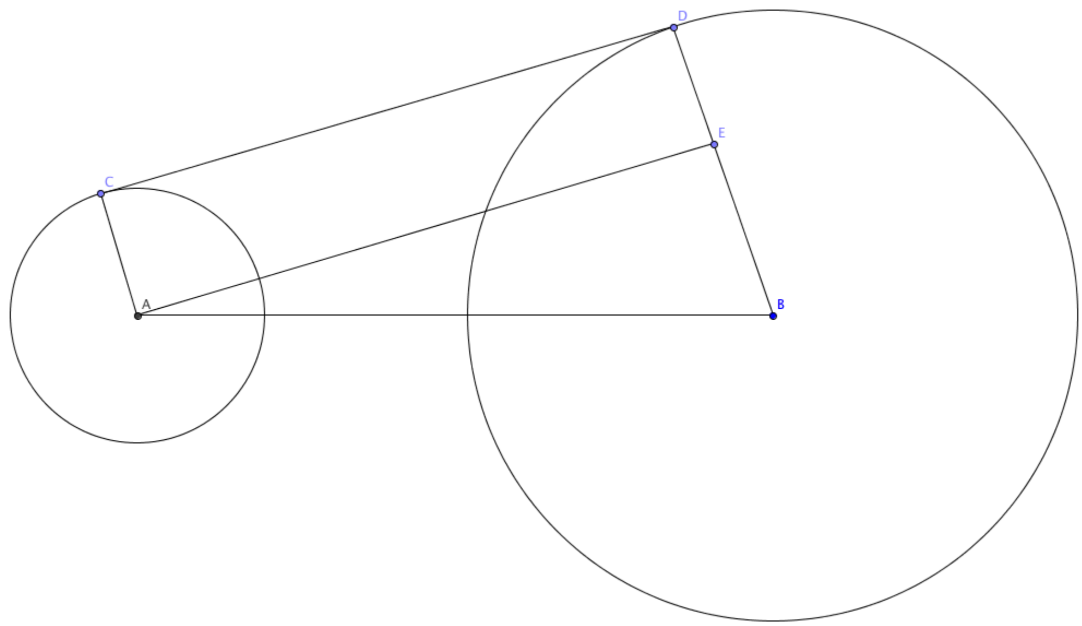
Question	Solution
P1.	Since $y = \frac{-15x+100}{3}$, the slope is $-\frac{15}{3} = \mathbf{-5}$.
P2.	By inspection, $n = \mathbf{2}$.
P3.	We have $x = 180 - 134 = \mathbf{46}$.
P4.	We have $2\ln e^{2013} = 2(2013)\ln e = \mathbf{4026}$.
P5.	We have $D(A+5) + BC = (4026)(-5+5) + (2)(46) = \mathbf{92}$.
1.	Rewrite as $\sqrt{10-x} - \sqrt{4-x} = 2$, then square both sides and simplify to obtain $5 - x = \sqrt{(10-x)(4-x)}$. Square both sides again and simplify and get $25 - 10x + x^2 = 40 - 14x + x^2$, or $4x = 15$, thus $x = \mathbf{15/4}$.
2.	Let x equal the side length of the octagon. By the Distance Formula, AD has length 5. Erect perpendiculars from B and C to AD, creating two sets of 45-45-90 triangles. Notice that AD can be expressed as $\frac{x}{\sqrt{2}} + x + \frac{x}{\sqrt{2}} = x + x\sqrt{2}$. Thus, $x + x\sqrt{2} = 5$, or $x = 5\sqrt{2} - 5$. The perimeter of the octagon is $8x = \mathbf{40\sqrt{2} - 40}$.
3.	By the Binomial Theorem, the term with degree 9 is $\binom{11}{9}(-4)^2(-x)^9 = \binom{11}{2}(-4)^2(-x)^9 = -(55)(16)x^9$, so the coefficient is $\mathbf{-880}$.
4.	Changing all the bases to base 2 yields $\log_2(2x) + \frac{1}{2}\log_2 x + \frac{1}{3}\log_2 x = 12$, or $\log_2\left(2x \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{3}}\right) = \log_2(2x^{\frac{11}{6}}) = 12$. In Exponential Form this is $2^{12} = 2x^{\frac{11}{6}}$, or $x = 2^6 = \mathbf{64}$.

5.	Note that if $B = 40\sqrt{2} - 40$, then $(B+40)^2 = B^2 + 80B + 1600 = \left(40\sqrt{2}\right)^2 = 3200$, so that $B^2 + 80B = 1600$. Therefore, we have $\frac{3(B^2+80B)C}{80AD} = \frac{3(1600)(-880)}{(80)(\frac{15}{4})(64)} = \mathbf{-220}$.
6.	Perfect squares have an odd number of positive divisors, hence it is for those values that the Tau Function will be congruent to 1 in modulo 2. The set of positive integer perfect squares less than 200 is $\{1^2, 2^2, \ldots, 14^2\}$, having **14** elements.
7.	In standard form, the ellipse has equation $\frac{(x+20)^2}{18} + \frac{(y-13)^2}{50} = 1$. The ellipse has major and minor semi-axes of $\sqrt{18}$ and $\sqrt{50}$, making the area $\pi\sqrt{(18)(50)} = \mathbf{30\pi}$.
8.	Working through the recursion formula backwards, we have $a_4 = \frac{a_5-3}{2} = \frac{-35-3}{2} = -19$, $a_3 = \frac{a_4-3}{2} = \frac{-19-3}{2} = -11$, …, eventually leading to $a_1 = \mathbf{-5}$.
9.	By the Remainder Theorem, the desired value is simply the polynomial evaluated at $x = -1$, or $-2 - 3 + 10 + 6 = \mathbf{11}$.
10.	We have $\frac{(A-C+D)\pi}{B} = \frac{(14-(-5)+11)\pi}{30\pi} = \frac{30\pi}{30\pi} = \mathbf{1}$.
11.	The polynomial factors as $(2x+1)(x^2-4) = 0$, so the sum of the roots is $\mathbf{-\frac{1}{2}}$. Note that this is the same value obtained via the "$-b/a$" trick, only because there are no imaginary roots.
12.	Consecutive integers imply that the sequence is arithmetic with common difference 1. We have $a_3 + a_4 = a_3 + (a_3 + 1) = 47$, so $a_3 = 23$. By extension, $a_1 = 21$ and $a_{10} = 30$. The sum of the first ten terms is

	$\frac{10}{2}(21+30) = \mathbf{255}$.
13.	If $P(x)$ does not have two distinct real roots that means the discriminant must be less than or equal to 0. So we have $(4p)^2 - 4(4)(4-3p) \leq 0$, or after some simplifying, $(p-1)(p+4) \leq 0$. The solution set to this inequality is $p \in [-4, 1]$, and the product of the negative integers in this interval is $(-4)(-3)(-2)(-1) = \mathbf{24}$.
14.	The volume of a regular octahedron with edge length s is $V = \frac{\sqrt{2}}{3}s^3$. Setting this equal to 4/3 yields $s = \sqrt{2}$. The surface area of a regular octahedron with side length s is $S = 2s^2\sqrt{3}$. Thus, the answer is $2\left(\sqrt{2}\right)^2\sqrt{3} = \mathbf{4\sqrt{3}}$.
15.	We have $\sqrt{B - \frac{AD^2}{C}} = \sqrt{255 - (-\frac{1}{2})(48)/24} = \sqrt{256} = \mathbf{16}$.
16.	If $\left\|\|x\| - 7\right\| \leq 8$, then $-8 \leq \|x\| - 7 \leq 8$, or $\|x\| \leq 15$, or $-15 \leq x \leq 15$ since the absolute value of any number must be at least 0. There are **31** integers in this interval.
17.	By the distance formula, $d^2 = \Delta x^2 + \Delta y^2$. The set of all possible Δx and Δy are $\{0, 1, 2, 3\}$ and $\{0, 1, 2\}$, respectively. The set of all possible nonzero values of d^2 is then $D = \{1, 4, 9, 5, 2, 8, 10, 13\}$. An irrational value for the distance will be formed for any irrational elements of D. The desired ratio is $\mathbf{5/8}$.
18.	Transform the equation into the diagram: *|**|**|**. Notice that there is a one-to-one correspondence between the diagram and a solution to the equation. Thus number of solutions is equal to the number of ways to arrange the diagram, which is $\binom{7+4-1}{4-1} = \binom{10}{3} = \mathbf{120}$.

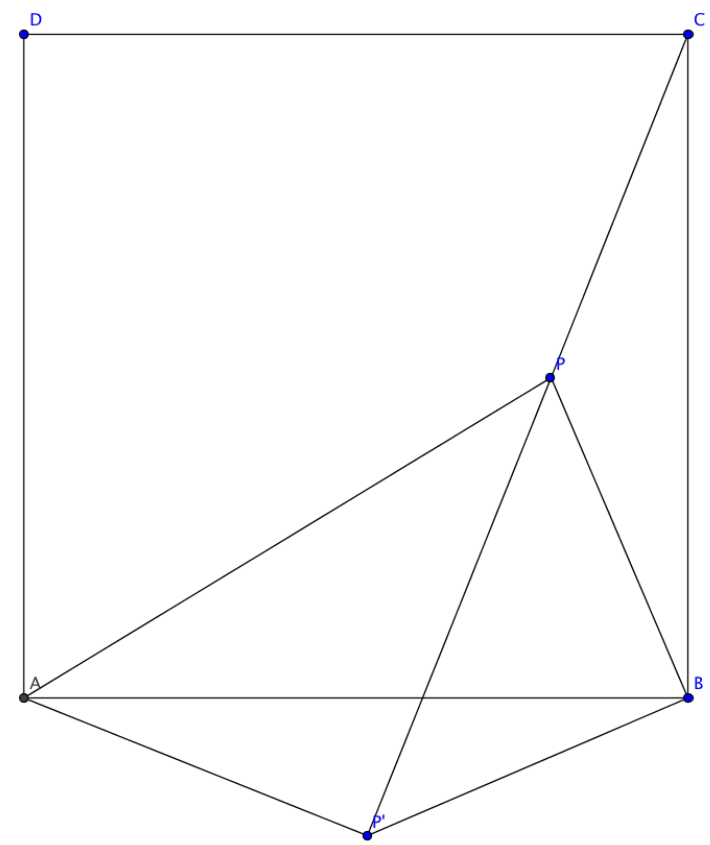
19.	There are six complex solutions to the equation, consisting of three conjugate pairs. The "principal" solution, $2e^{\frac{\pi}{6}i}$, has a positive real part and therefore, so will its conjugate. The product of two complex conjugates is the norm-square, so the answer is $\left\|2e^{\frac{\pi}{6}i}\right\|^2 = 2^2 = \mathbf{4}$.
20.	We have $A + \frac{C}{BD} = 31 + \frac{120}{\left(\frac{5}{8}\right)(4)} = \mathbf{79}$.
21.	Notice that $39^2 + 52^2 = 25^2 + 60^2$. Thus, the quadrilateral in question consists of two right triangles glued together at their hypotenuse. The area is therefore equal to $\frac{(39)(52)+(25)(60)}{2} = \mathbf{1764}$.
22.	If $S = -\frac{2}{5} + \frac{5}{25} + \cdots$, then $\frac{S}{5} = -\frac{2}{25} + \frac{5}{125} + \cdots$, so that $S - \frac{S}{5} = -\frac{2}{5} + \frac{7}{25} + \frac{7}{125} + \cdots$. Notice that starting from the second term on, the series is geometric. Therefore, $S - \frac{S}{5} = \frac{4S}{5} = -\frac{2}{5} + \frac{\frac{7}{25}}{1-\frac{1}{5}} = -\frac{1}{20}$, making $S = \mathbf{-1/16}$.
23.	Since $1352 = 2^3 \cdot 13^2$, the sum of the positive divisors is $(1+2+4+8)(1+13+169) = \mathbf{2745}$.
24.	Guess-and-check yields $M = 512 = 8^3$ and $N = 343 = 7^3$, so the answer is **15**.
25.	We have $(A - C)(B^{-1} + D) = (1764 - 2745)\left(\left(-\frac{1}{16}\right)^{-1} + 15\right) = \mathbf{981}$.
26.	Observe that M is a sort of permutation-scaling matrix, where the first element goes to the fourth position and gets scaled by $\frac{1}{4}$, the second element goes to the first position without scaling, etc. Using this reasoning, we can go backwards and deduce that $M^{-1} \times \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 4d \\ a \\ 2b \\ \frac{c}{3} \end{bmatrix}$, making $M^{-1} = \begin{bmatrix} 0 & 0 & 0 & 4 \\ 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 \end{bmatrix}$, so the sum

	of the elements of $3M^{-1}$ is $3\left(4+1+2+\frac{1}{3}\right)=\mathbf{22}$.
27.	The sum of the roots of f is $-\frac{-3}{2}=\frac{3}{2}$, so that means the third root is $\frac{1}{2}$. Thus, the factorization of f is $f(x)=(2x-1)(x-3)(x+2)=2x^3-3x^2-11x+6$, making $\lvert a\rvert+\lvert b\rvert=\lvert -11\rvert+\lvert 6\rvert=\mathbf{17}$.
28.	If $\log_9 x=\log_{12} y=\log_{16}(x+y)=M$, then $9^M=x$, $12^M=y$ and $16^M=x+y$, so $9^M+12^M=16^M$ or after dividing both sides by 9^M and simplifying, $1+\left(\frac{4}{3}\right)^M=\left(\left(\frac{4}{3}\right)^M\right)^2$. Note that $N=\frac{y}{x}=\frac{12^M}{9^M}=\left(\frac{4}{3}\right)^M$. Thus, the equation becomes $1+N=N^2$, so that $N^2-N=\mathbf{1}$.
29.	The parabola in standard form is $-8(x+2)=(y-3)^2$, so the vertex is at $(-2,3)$ and $p=\left\lvert-\frac{8}{4}\right\rvert=2$. Since this is a left-facing parabola, the coordinates of the focus is $(-2-2,3)=(-4,3)$, and its distance from the origin is $\sqrt{(-4)^2+3^2}=\mathbf{5}$.
30.	We have $(B-C)^2+(A+D)^{\frac{2}{3}}=(17-1)^2+(22+5)^{\frac{2}{3}}=256+9=\mathbf{265}$.
31.	Suppose P has degree n. The left-hand side of the equation has degree $2n$ while the right-hand side has degree of $n+1$. Therefore, $n=1$ and P is a linear function, say $P(x)=mx+b$. Substitute this into the equation to obtain $mx^2+b+2x^2+10x=2x(m(x+1)+b)+3$, and combine like-coefficients to get $(m+2)x^2+10x+b=2mx^2+(2b+2m)x+3$. Setting corresponding coefficients to each other yields $m=2$ and $b=3$, so $P(x)=2x+3$ and $P(100)=2(100)+3=\mathbf{203}$.
32.	We have $4^{5x-3}=64^{7x+1}$, or $2^{2(5x-3)}=2^{6(7x+1)}$. Setting exponents equal to each other yields $2(5x-3)=6(7x+1)$, or $x=\mathbf{-3/8}$.

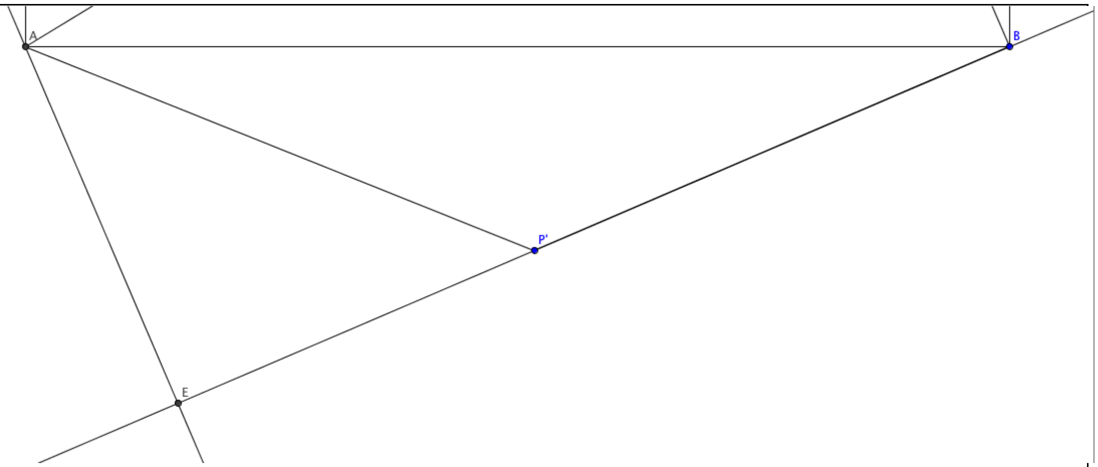
33.	 Refer to the diagram above. We see that the external tangent CD has the same length as AE, which is the leg of right triangle AEB. By the Pythagorean Theorem, $AE = \sqrt{AB^2 - BE^2} = \sqrt{AB^2 - (BD - DE)^2} = \sqrt{25^2 - 7^2} = \mathbf{24}$.
34.	Note that $a_{32} = 16 + (16 \times 2)^2 = 16\big(1 + (64)\big) = (16)(65)$ and $a_{33} = 16 + (2 \times 16 + 1)^2 = 4(16)^2 + 16 + 4(16) + 1 = (16 + 1)(4 \times 16 + 1) = (17)(65)$, so we suspect that **65** is the desired maximum GCD. Note that $(3 + 2n)a_n + (1 - 2n)a_{n+1} = 65$ for all n; therefore, any GCD of a_n and a_{n+1} will be a multiple of 65. Thus, 65 is the maximum GCD indeed.
35.	We have $A + BC + D = 203 + \left(-\frac{3}{8}\right)(24) + 65 = \mathbf{259}$.
36.	Let P equal the intersection of the medians BE and AD. Point P divides the medians into a 2:1 ratio, so AP = 4 and EP = 3. Triangle APE is a right triangle with area 6, which happens to be one-sixth the area of ABC. The answer is **36.**
37.	If $8100 = 108^a 45^b 50^c$, then $2^2 \times 3^4 \times 5^2 = (2^2 \times 3^3)^a (3^2 \times 5)^b (2 \times 5^2)^c$, or $2^2 \times 3^4 \times 5^2 = 2^{2a+c} \times 3^{3a+2b} \times 5^{b+2c}$, resulting in the system: $2a + c = 2$,

	$3a + 2b = 4$, and $b + 2c = 2$. Use your favorite system-solving technique to obtain $b = \mathbf{10/11}$.
38.	The desired count is just the number of positive integral factors of $100^2 = 2^4 5^4$, or $(5)(5) = \mathbf{25}$.
39.	The first few terms of the sequence are $a_1 = 2$, $a_2 = 1 - \frac{1}{a_1} = 1 - \frac{1}{2} = \frac{1}{2}$, $a_3 = 1 - \frac{1}{a_2} = 1 - \frac{1}{\frac{1}{2}} = -1$, and this cycle continues onward. Every three terms starting from the first will have a sum of 1.5, and since $833 = 3(277) + 2$, the desired sum is $277(1.5) + 2 + \frac{1}{2} = \mathbf{418}$.
40.	We have $BD + A - C = \left(\frac{10}{11}\right)(418) + 36 - 25 = \mathbf{391}$.
41.	If $S = \{1\}$, then $\sum_{n=1}^{1} \frac{1}{\Pi(S_n)} = 1$. If $S = \{1, 2\}$, then $\sum_{n=1}^{3} \frac{1}{\Pi(S_n)} = 2$. In general, it can be proven by induction that if $S = \{1, 2, 3, \dots, n\}$, then $\sum_{n=1}^{2^n - 1} \frac{1}{\Pi(S_n)} = n$, so for this problem the answer is **4**.
42.	We know that $x + 18 + 4 + 13 + 6 = 50$, making $x = \mathbf{9}$. By coincidence, this is also the median.
43.	The primes are 53 and 59, and they have an arithmetic mean of 56 and half positive difference of 3. Therefore, $\frac{1}{4}(b^2 - a^2) = \frac{b-a}{2} \cdot \frac{b+a}{2} = 3(56) = \mathbf{168}$.

44.



Refer to the diagram above. Take triangle CPB and rotate it across B so that C coincides with A, resulting in triangle AP'B. We have PB = P'B and $m\angle \text{CBP} = m\angle \text{ABP}'$, so triangle PBP' is a 45-45-90 triangle, making $\text{PP}' = (2\sqrt{2})\sqrt{2} = 4$. Consequently, triangle APP' is a 3-4-5 right triangle, where ∠PP'A is the right angle. To obtain the area of square ABCD, we only need to find the value of AB^2. Extend segment P'B into a line and drop altitude AE, as shown below.

	 We have $\mathrm{CP} = \mathrm{AP}' = 3$ and since $AP'E$ is a 45-45-90 triangle, $AE = EP' = 3/\sqrt{2}$. Thus, by the Pythagorean Theorem, $AB^2 = \mathrm{AE}^2 + \mathrm{EB}^2 = \left(\frac{3}{\sqrt{2}}\right)^2 + \left(\frac{3}{\sqrt{2}} + 2\sqrt{2}\right)^2 = \mathbf{29}$.
45.	We have $\sqrt{A} - \sqrt{B} + \sqrt{C+1} - \sqrt{D-4} = \sqrt{4} - \sqrt{9} + \sqrt{168+1} - \sqrt{29-4} = \mathbf{7}$.
46.	Since $2(3n+5) - 3(2n+3) = 1$, $3n+5$ and $2n+3$ are relatively prime. Also, since $2210 = 2 \times 5 \times 13 \times 17$, we have $\frac{2210}{(3n+5)(2n+3)} = \frac{2\times5\times13\times17}{(3n+5)(2n+3)} = 5\frac{(2\times13)(17)}{(3n+5)(2n+3)}$. Setting $3n+5 = 26$ and $2n+3 = 17$ yields $n = \mathbf{7}$. Turns out this is the only valid value for n.
47.	Add the two linear equations to obtain $5x = 20$, or $x = 4$, making $y = 2$. Thus, we have $x^2 + y^2 = 16 + 4 = \mathbf{20}$.
48.	There are 10 possible units digits: 0, 1, .., 9. By the Pigeonhole Principle, we need at least **11** numbers in the collection.
49.	Since $25^2 < 650 < 26^2$, f will take on every integer from 1 to 25, inclusive. We find that f equals 1 for two values (1 and 2), f equals 2 for four values (3,

	4, 5, and 6), f equals 3 for six values, etc. Based on this pattern, we have $\sum_{n=1}^{650}\frac{1}{f(n)} = 2(1) + 4\left(\frac{1}{2}\right) + 6\left(\frac{1}{3}\right) + \cdots 50\left(\frac{1}{25}\right) = 2 + 2 + 2 + \cdots + 2 = 25(2) =$ **50**.
50.	We have $(A+B)^{\frac{1}{3}} + (C+D+3)^{\frac{1}{2}} = (7+20)^{\frac{1}{3}} + (11+50+3)^{\frac{1}{2}} = 3 + 8 =$ **11**.