The abbreviation NOTA means "None of These Answers."

1. What is the area bounded by the graph of $f(x)=\sin x$ and $g(x)=\cos x$ between $x=\frac{\pi}{4}$ and $x=\frac{3\pi}{4}$.

A. $4\sqrt{2}$ B. $2\sqrt{2}$
C. $\sqrt{2}$ D. $\frac{\sqrt{2}}{2}$
E. NOTA

2. The area represented by the definite integral $\int_0^5 |3x-3|dx$ can also be represented geometrically, as the area of triangles, by which expression?

A. $\frac{1}{2}(0)3+\frac{1}{2}(5)12$
B. $\frac{1}{2}(5 \bullet 12)$
C. $\frac{1}{2}(4 \bullet 12)-\frac{1}{2}(1 \bullet 3)$
D. $\frac{1}{2}(1 \bullet 3)+\frac{1}{2}(4 \bullet 12)$
E. NOTA

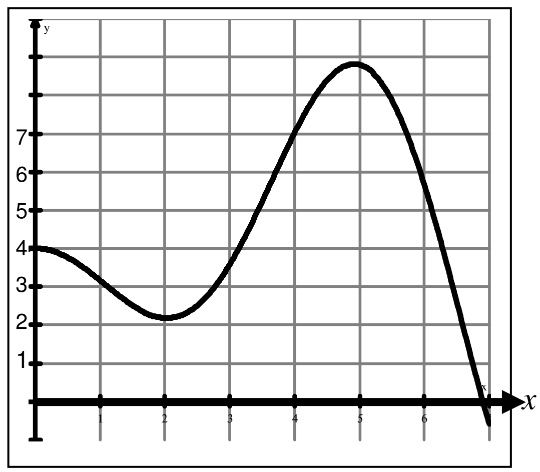
3. A rock is thrown into a lake and where it enters the water, a circular ripple is created. The circle expands so that its circumference is increasing at 12π cm per second. At what rate, in cm per second, is the radius of the circle changing when the circumference is 20π cm?

A. $\frac{3}{10\pi}$ B. $\frac{3}{5\pi}$ C. 6
D. 30 E. NOTA

4. The area bounded by $y=f(x)$ and the x-axis between $x=a$ and $x=b$ is $4a-b$. The graph of f is continuous for all values of x and $f(x)>0$ for all x, and $a<b$. Find the area bounded by $y=f(x)+4$ between $x=a$ and $x=b$.

A. $4a-b-4$ B. $4a-b-4$
C. $8a+3b$ D. $3b$
E. NOTA

5.



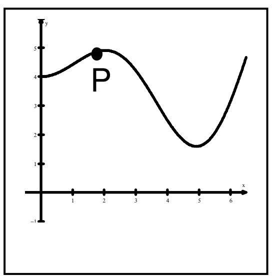
During a 7-hour day, it rains off and on, and the rate of change of area of the surface of a puddle is given by $A(t)$. The graph of $A(t)$ is shown above, over the window (horizontal axis) [0, 7] hours and (vertical axis) [-1, 10] sq. inches per hour. At $t=0$, the puddle has surface area 2 sq. inches. Use a trapezoidal approximation with subdivisions [0,2], [2,4], [4,5], [5, 7] and give the approximate surface area of water in the puddle at $t=7$ hours. The area is between __ and __ sq inches.

A. 29 and 30 B. 31 and 32
C. 32 and 33 D. 33 and 34
E. NOTA

6. The graph of f is continuous for all x; $f(x)<0$ over [0, 5) and $f(x)>0$ over (5, 10].
$A=\int_0^5 f(x)dx = 2-\int_5^{10} f(x)dx$ then which is the area bounded by the graph of f and the x-axis, between the lines x=0 and x=10?

A. $2A+2$
B. $2-2A$
C. $2-A$
D. 2
E. NOTA

7. The graph of the surface area of a balloon, as a function of time, is shown as the heat and pressure of a room cause its volume to fluctuate. Which is true about the surface area of the balloon at time 1.9 (indicated by the point P)?



A. The surface area is increasing at a decreasing rate.
B. The surface area is increasing at an increasing rate.
C. The surface area is decreasing at an increasing rate.
D. The surface area is decreasing at a decreasing rate.
E. NOTA

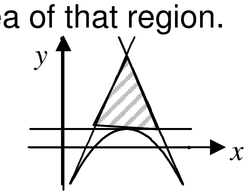
8. The graph shown in question #7 contains the point (2, 5). The balloon is always spherical. At t=2, the numerical rate of change of surface area of the balloon with respect to time varies directly with the rate of change of the volume of the balloon with respect to time, discounting all units. The constant of variation is $\frac{A\sqrt{B\pi}}{B}$. Give the value of $A+B$.

A. 6 B. 7
C. 8 D. 9
E. NOTA

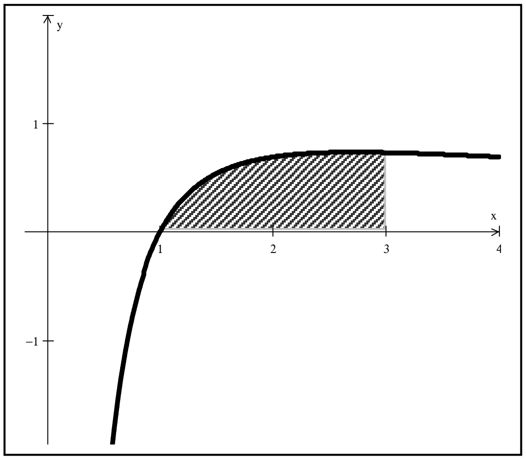
9. A circle is inscribed in an equilateral triangle. The triangle's area is increasing at $2\sqrt{3}$ sq cm per minute. At what rate in sq cm per minute is the circle's area increasing when the circle's radius is 3 cm?

A. $\frac{1}{9\pi}$ B. $\frac{\pi\sqrt{3}}{6}$
C. $\frac{\pi\sqrt{3}}{3}$ D. $\frac{2\pi}{3}$
E. NOTA

10. The three lines tangent to the graph of $y=-x^2+4x-3$ at x=0, x=2 and x=4 bound a triangular region. Find the area of that region.



A. 8 B. 6
C. 4 D. 2
E. NOTA



The graph of $f(x)=\dfrac{\ln(x^2)}{x}$ over (0, 4] is shown above, and the region R bounded by the x-axis, the graph of f and x=3 is shaded. **Use this information to answer questions 11 and 12.**

11. Which gives the volume of the solid that has base R and which has cross sections perpendicular to the x-axis which are semicircles?

A. $\dfrac{\pi}{2}\int_1^3\left(\dfrac{\ln\left(x^2\right)}{x}\right)dx$

B. $\dfrac{\pi}{4}\int_1^3\left(\dfrac{\ln x}{x}\right)^2 dx$

C. $\dfrac{\pi}{4}\int_1^3\left(3-\dfrac{\ln x^2}{x}\right)^2 dx$

D. $\dfrac{\pi}{2}\int_1^3\left(3-\dfrac{\ln\left(x^2\right)}{x}\right)^2 dx$

E. NOTA

12. (Refer to the information above question #11.) What is the area of region R?

A. $4\ln 2$ B. $(\ln 3)^2$

C. $\left(\ln\dfrac{3}{2}\right)^2$ D. $\ln 9$

E. NOTA

13. A right triangle has one leg increasing at 2 cm/min and the other leg decreasing at $\dfrac{1}{3}$ cm/min. When the area of the triangle is increasing at 10 sq cm/min and the legs are equal, what is the length of the hypotenuse in cm?

A. $12\sqrt{2}$
B. 12
C. $6\sqrt{2}$
D. $4\sqrt{6}$
E. NOTA

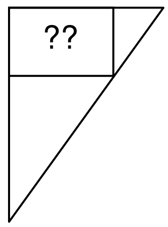
14. A triangle has sides 12 cm, 10 cm and 8 cm. The angle included in the 12 cm and 10 cm sides remains the same measure, while the 12 cm and 10 cm sides both increase at 1 cm per second. At what rate is the area of the triangle changing in sq cm per second, 3 seconds after the original 12-10-8 lengths are observed?

A. $\dfrac{56}{5}$ B. $\dfrac{44}{5}$

C. $\dfrac{11\sqrt{7}}{2}$ D. $\dfrac{7\sqrt{7}}{2}$

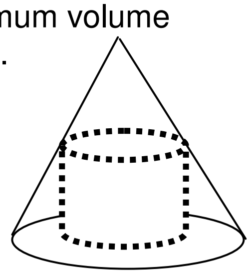
E. NOTA

15. You need to fence in a rectangular playground for the neighborhood children, to fit into a right-triangular plot with triangle legs measuring 4 meters and 12 meters. Two sides of the play zone will lie on the legs of the triangle as shown. What is the maximum area in sq meters for this play zone?



A. 12 B. $14\sqrt{2}$
C. 16 D. $8\sqrt{3}$
E. NOTA

16. A right circular cylinder is inscribed in a right circular cone so that the center axis of each coincide. The cone has a height of 8 ft and a radius of 6 feet. Find the radius in feet of the cylinder with the maximum volume that can fit in this cone.



A. 1 B. 2
C. 3 D. 4
E. NOTA

17. Region R is bounded by the graphs of $y = x^2$ and the line y=1. The volume of the resultant solid if R is revolved about the line y=2 is $\frac{N}{D}\pi$ for $\frac{N}{D}$ a fraction in simplest form. Give the value of $N+D$.
A. 71 B. 43
C. 28 D. 17
E. NOTA

18. Which gives the area of the region bounded by the graphs of $y = e^{2x}$, $y = 2$ and $x = 0$?

A. $\pi\int_0^2 (4 - e^{4x})dx$

B. $\int_0^2 (2 - e^{2x})dx$

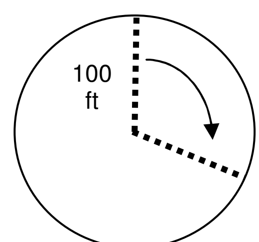
C. $\int_0^2 \sqrt{\ln y}\, dy$

D. $\int_1^2 \ln\sqrt{y}\, dy$

E. NOTA

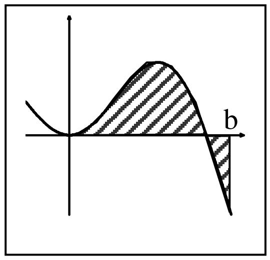
19. A solid has base on the xy-plane defined by the ellipse with equation $x^2 + 4y^2 = 4$, and cross-sections of the solid perpendicular to the x-axis are squares. What is the volume of the solid?
A. $\frac{16}{3}$ B. $\frac{8}{3}$
C. $\frac{4}{3}$ D. $\frac{\pi}{6}$
E. NOTA

20. A camera rotates at 0.001 radians per minute around a fixed axis. The camera has a range of 100 feet. In 15 minutes, tell the area (ground area) that the camera has scanned, in square feet.



A. 150π B. 100π
C. 150 D. 75
E. NOTA

21. The graph of $y = f(x)$ is shown below with zeros at x=0 and x=a. The shaded region is between the x-axis, the graph of f and the line x=b. Which expression does NOT represent the area of the shaded region?



A. $\int_0^b |f(x)|\,dx$

B. $\left|\int_0^a f(x)dx + \int_b^a f(x)dx\right|$

C. $\int_0^a f(x)dx + \int_b^a f(x)dx$

D. $\int_0^a -f(x)dx - \int_a^b |f(x)|dx$

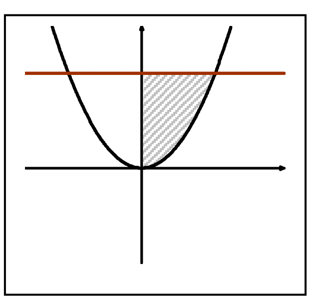
E. NOTA

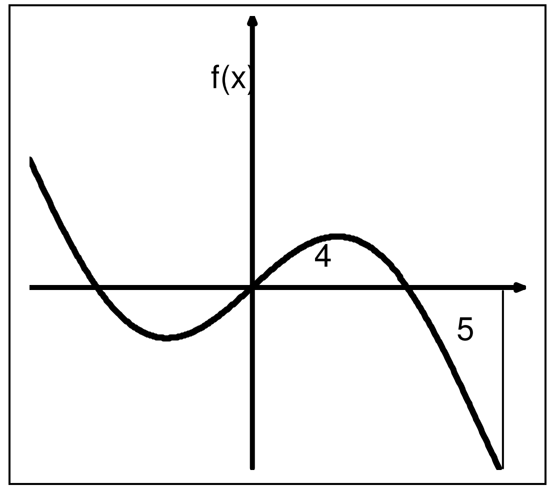
22. Region R is enclosed by the graph of $y = \ln(x+1)$, the y-axis and the line y=1, What is the area of region R?

A. $e-2$
B. $e-1$
C. e
D. $2e$
E. NOTA

23. The area in Quadrant I that is enclosed by the graph of $y = x^2$ and the line $y = k$ is $\frac{16}{3}$, for $k > 0$. Give the value of k.

A. 8
B. 6
C. 4
D. 2
E. NOTA





The graph of an odd function $y = f(x)$ is shown over [-6, 6] above. f has zeros at x=0 and x=± 3 and the area enclosed by f and the x-axis between x=0 and x=3 is 4; the area enclosed by f and the x-axis between x=3 and x=6 is 5.

Use the information above to answer questions 24-25.

24. What is the area enclosed by the graph of $y = f(|x|)$ and the x-axis, between x=−6 and x=6 ?

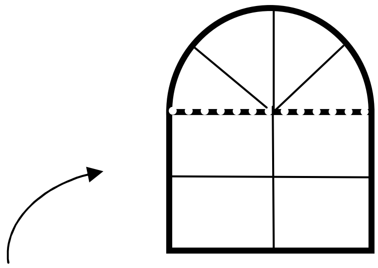
A. 0 B. 2 C. 9
D. 18 E. NOTA

25. Give the value of $\int_{-6}^{6} f(|x|)dx$.

A. 0 B. -2
C. 18 D. 8
E. NOTA

26. A triangle has base a constant 10 cm and the height to that base is decreasing by 0.5 cm per minute. Find the rate that the area is decreasing in sq cm per minute, when the height is 5 cm.

A. 2.5 B.5
C. 7 D.8
E. NOTA

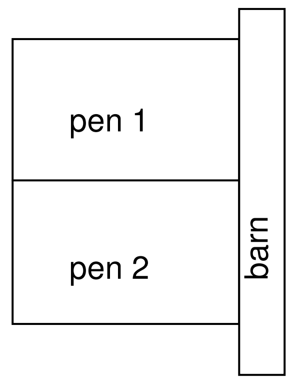


27. A Norman window has a semicircle on top of a rectangle as shown in the figure. Suppose there is an amount of $(8+\pi)$ ft of metal trim for all of the outer edges of the window (perimeter: three sides of the rectangle and arc of the semicircle). Find the dimensions of the window that will maximize the area of the window. The radius of the semicircle in feet for those dimensions. can be written as $\frac{\pi+X}{\pi+Y}$. Give the value of $\frac{X}{Y}$.

A. 1
B. 2
C. 3
D. 4
E. NOTA

28. The area bounded by $f(x)=kx^2$ (for k a positive constant) and $g(x)=x^3$ is twice the area bounded by $h(x)=x^2$ and $m(x)=kx^3$. Find the value of k.

A. 64
B. 32
C. $\sqrt[6]{2}$
D. $\sqrt[7]{2}$
E. NOTA

29. 

A farmer intends to build two pens for his horses. One side will have a barn attached so that side will need no fence. The farmer will use 1200 feet of wire. The wire will make three parallel sides as shown and two sides opposite the barn. If the farmer maximizes the area of the pens and the area of each pen is equal, tell the area of one pen in sq feet.

A. 32000 B. 42000
C. 60000 D. 55000
E. NOTA

30. The definite integral

$$\pi\int_{-3}^{3}\left(\sqrt{9-x^2}\right)^2 dx$$

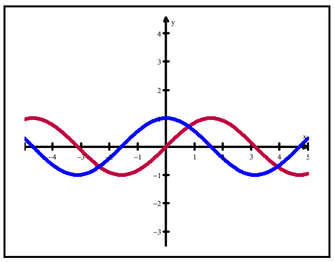
represents the volume of a sphere. What is the surface area of the sphere?

A. 9π B. 12π
C. 18π D. 36π
E. NOTA

1. C	11. E	21. D
2. D	12. B	22. A
3. C	13. A	23. C
4. D	14. D	24. D
5. D	15. A	25. B
6. B	16. D	26. A
7. A	17. A	27. B
8. D	18. D	28. D
9. D	19. E	29. C
10. C	20. D	30. D

Solutions:

1.

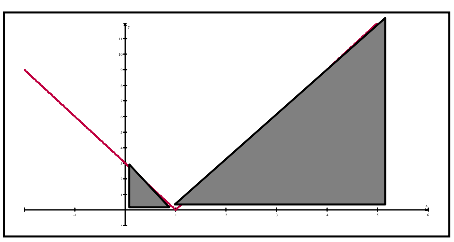


$$\int_{\pi/4}^{3\pi/4} (\sin x - \cos x)dx =$$

$$(-\cos\frac{3\pi}{4} - \sin\frac{3\pi}{4}) - \cos\frac{\pi}{4} + \sin\frac{\pi}{4} =$$

$\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2}$. Choice C

2.



The regions are triangles with bases and heights 1 and 3, and 4 and f(5)=12, respectively. So 1/2*1*3+ 1/2*4*12 is choice D.

3. $\frac{dC}{dt} = 2\pi\frac{dr}{dt}$ from the circumf of a circle formula. $12\pi = 2\pi\frac{dr}{dt}$ so dr/dt=6. Choice C.

4. $\int_a^b f(x)dx = 4a - b$ so raising the graph 4 units adds an area of a rectangle with height 4 and base (b-a). $4a - b + 4b - 4a$ =3b. Choice D.

5. Approximated: $\frac{1}{2}*2*(4+2.2)+$ $\frac{1}{2}*2*(2.2+7)+$ $\frac{1}{2}*1*(7+8.8)+$ $\frac{1}{2}*2*(8.8+-0.5)$ = 31.6. Add the initial 2 to get 33.6. Choice D.

6. A must be negative since f is below the x-axis and $A = \int_0^5 f(x)dx$. So the area over [0, 5] is –A, a positive value. Since $2 - \int_5^{10} f(x)dx$ = A we solve to get the value over [5, 10] is 2-A which is also a positive value. Add 2-A + -A to get area 2-2A. This is a positive value. Choice B.

7. The graph shown is increasing but is concave down so the rate of increase is decreasing. That is, the SA is increasing at a decreasing rate. Choice A.

8. Time=2 gives SA=5. $5 = 4\pi r^2$ so $r = \sqrt{\frac{5}{4\pi}}$. $\frac{dS}{dt} = 8\pi r\frac{dr}{dt}$ → $\frac{dS}{dt} = 8\pi\frac{\sqrt{5}}{2\sqrt{\pi}}\frac{dr}{dt}$. $\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$ → $\frac{dV}{dt} = 5\frac{dr}{dt}$, $\frac{dS}{dt} = 8\pi\frac{\sqrt{5}}{2\sqrt{\pi}}\left(\frac{1}{5}\bullet\frac{dV}{dt}\right)$

$\frac{dS}{dt} = \frac{4\sqrt{5\pi}}{5}\frac{dV}{dt}$. A=4 and B=5. A+B=9, choice D.

9. Since area of an equilateral triangle is $\frac{\sqrt{3}}{4}side^2$ and the side of the triangle is $2r\sqrt{3}$ for radius r, the area

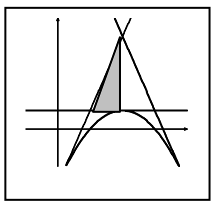
of the triangle is $\frac{\sqrt{3}}{4}\left(2r\sqrt{3}\right)^2 =$

$3\sqrt{3}r^2$. So $\frac{dA}{dt} = 6\sqrt{3} * r\frac{dr}{dt}$

$2\sqrt{3} = 6\sqrt{3} * 3\frac{dr}{dt}$.. dr/dt=1/9. The area of the circle will change with

$2\pi r\frac{dr}{dt} = 2\pi(3)\frac{1}{9} = \frac{2}{3}\pi$. Choice D.

10. dy/dx= -2x+4 and the tangent line at x=0 has equation y= 4x-3. The line at x=2 is horizontal with equation y=1. Consider half of the area desired. Let 4x-3=1 gives that the two mentioned tangents meet at (1, 1). So the shaded triangle to the right has base 1 and height at (2, 5) for h=4. A=1/2*1*4. Double that to get area 4. Choice C.



11. The diameter of a cross-section is $\frac{\ln x^2}{x} = \frac{2\ln x}{x}$ and radius is (lnx)/x.

Area of a cross section is $\frac{\pi}{2}\left(\frac{\ln x}{x}\right)^2$

so the volume is the integral of this, for x=1 to x=3. Choice E.

12. $\int_1^3 \left(\frac{\ln\left(x^2\right)}{x}\right)dx = \int_1^3 \left(\frac{2\ln(x)}{x}\right)dx$

$\int_0^{\ln 3} 2u\ du = u^2 \Big|_0^{\ln 3} = \left(\ln 3\right)^2$. Choice B

13. $A = \frac{1}{2}bh$. $\frac{dA}{dt} = \frac{1}{2}h\frac{db}{dt} + \frac{1}{2}b\frac{dh}{dt}$.

$10 = \frac{1}{2}h(2) + \frac{1}{2}h(-\frac{1}{3})$. $20 = \frac{5}{3}h$

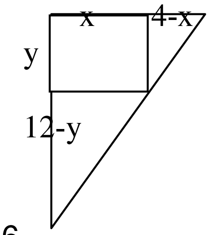
and 12=h so hypotenuse is $12\sqrt{2}$. Choice A.

14. $A = \frac{1}{2}abSinC$. $\cos C = -\frac{3}{4}$

$\frac{dA}{dt} = \frac{1}{2}\sin C\left(a\frac{db}{dt} + b\frac{da}{dt}\right)$.

$= \frac{1}{2}\left(\frac{\sqrt{7}}{4}\right)\left(15+13\right) = \frac{7\sqrt{7}}{2}$. Choice D

15. Let the rectangle have dimensions x and y.



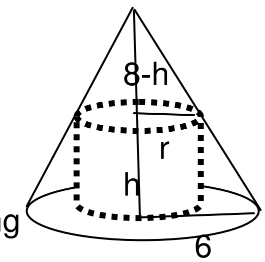
$\frac{4-x}{y} = \frac{4}{12}$. $12 - 3x = y$.

Area = xy = x(12-3x).

$A' = 12 - 6x = 0$ when x=2. y=6.

Area = 12. Choice A.

16. $\frac{8-h}{8} = \frac{r}{6}$ for r and h dimensions of the cylinder.



$V = \pi r^2(-\frac{4}{3}r + 8)$ using the proportion above to substitute.

$V = -\pi(\frac{4}{3}r^3 - 8r^2)$. V ' $= -\pi(4r^2 - 16r)$

which equals 0 at r=0 and r=4. For r=4, V" < 0 so this is the radius at the relative max. Choice D.

17. $2\pi\int_0^1 ((2-x^2)^2 - 1)dx =$

$2\pi\int_0^1 (3 - 4x^2 + x^4)dx =$

$2\pi\left(3x - \frac{4}{3}x^3 + \frac{1}{5}x^5\right)\Big|_0^1 = \frac{56}{15}\pi$.

56+15=71. Choice A.

18. Using y-variables, D gives the area. Choice A has π. Choice B has the wrong limits. Choice C has the wrong function. D is the answer.

19. $\int_{-2}^{2}\left(2\sqrt{1-\frac{1}{4}x^2}\right)^2 dx = 8\int_0^2\left(1-\frac{1}{4}x^2\right)dx =$

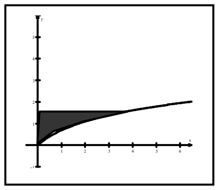
$8\left(x - \frac{1}{12}x^3\right)\Big|_0^2 = \frac{32}{3}$. Choice E.

20. Not really calculus: 15 min (0.001rad)/(2π) is the proportion of the circle's area, so multiply by $100^2\pi$ to get 75. Choice D

21. In choice A, $y = \left|f(x)\right|$ has equivalent area and is above the x-axis. In choice B, the second limits are

reversed so the second quantity is also positive. Same with C. D is negative and not area. This is the answer.

22. $\int_0^1 (e^y - 1)dy = e^y - y\Big|_0^1 =$ (e-1)-1=e-2 Choice A.



23. $\int_0^{\sqrt{k}} \left(k - x^2\right)dx$ =16/3.

$$kx - \frac{x^3}{3}\Big|_0^{\sqrt{k}} = k\sqrt{k} - \frac{1}{3}k\sqrt{k} = \frac{2}{3}k\sqrt{k} = \frac{16}{3}$$

$k\sqrt{k} = 8$; $k^{\frac{3}{2}} = 8, k = 8^{2/3}$ =4. Choice C

24. f(|x|) is unchanged for x>0 and for x<0 the graph is a reflection of the right side, over the y-axis. Area is twice 4+5, or 18. Choice D.

25. The integral gives -5 + 4 + 4 + -5 = -2. Choice B.

26. $A = \frac{1}{2}(10)h = 5h$. $\frac{dA}{dt} = 5\frac{dh}{dt}$. 5(-0.5) gives rate -0.5. The question asks for the rate of decrease so that is a positive 0.5. Choice A.

27. $\pi r + 2x + 2r = 8 + \pi$ using r as the radius of the semicircle and x as the vertical sides of the rectangle.

$$A = \frac{1}{2}\pi r^2 + 2xr =$$

$$\frac{1}{2}\pi r^2 + (8r + \pi r - \pi r^2 - 2r^2).$$

$$A' = \pi r + 8 + \pi - 2\pi - 4r = 0$$

$r = \dfrac{-8+\pi}{-4+\pi}$. -8/-4= 2. Choice B.

28. $y = x^3$ and $y = kx^2$ meet at x=k.

$y = kx^3$ and $y = x^2$ meet at x= 1/k.

$$\int_0^k \left(kx^2 - x^3\right)dx = 2\int_0^{1/k} \left(x^2 - kx^3\right)dx$$

$\dfrac{kx^3}{3} - \dfrac{x^4}{4}\Big|_0^k = \dfrac{k^4}{12}$ equals

$$2\left(\frac{x^3}{3} - \frac{kx^4}{4}\right)\Bigg|_0^{1/k} = 2\left(\frac{1}{3k^3} - \frac{1}{4k^3}\right)$$

$= \dfrac{1}{6k^3}$. So $\dfrac{k^4}{12} = \dfrac{1}{6k^3}$. k= $\sqrt[7]{2}$.

Choice D.

29. Let each horizontal segment be x and vertical be y. 2y+3x=1200. Area of pens= 2xy = x(1200-3x). A'=1200-6x=0 when x=200 and y=300 and area of one pen is 60000. Choice C.

30. The quantity squared comes from equation $y = \sqrt{9 - x^2}$ which is the top half of a circle of radius 3. The sphere would have surface area $4\pi(3)^2$, choice D.