1. C	6. E	11. A	16.C	21. A	26. B
2. B	7. A	12. D	17. D	22. C	27. A
3. B	8. A	13. B	18. A	23. C	28. E
4. B	9. B	14. E	19. B	24. C	29. E
5. C	10. A	15. A	20. D	25. B	30. C

1. C. Using basic properties of arithmetic sequences, we find $a_n = 3n+1$ and $b_m = 2m+5$. We need to find integer solutions to $3n+1 = 2m+5$. Taking both sides $\bmod 3$, we obtain that $m \equiv 1 \bmod 3$, or that $m = 3k+1$ for integers k. Then the numbers in both b_m and a_n are of the form $2(3k+1)+5 = 6k+7$. Finally, $6k+7 < 1000 \to < 165.5$, so there are $\boxed{165}$ common elements.

2. B. Recall that $T_n = \frac{n(n+1)}{2}$. So the sum we seek is then $\sum_{n=1}^{\infty} \frac{n(n+1)}{2^n} = \sum_{n=1}^{\infty} \frac{n^2}{2^n} + \sum_{n=1}^{\infty} \frac{n}{2^n}$.

Let S denote $\sum_{n=1}^{\infty} \frac{n}{2^n}$. Then $\frac{S}{2} = \sum_{n=1}^{\infty} \frac{n}{2^{n+1}}$. Subtracting these two values yields $\frac{S}{2} = \sum_{n=1}^{\infty} \frac{1}{2^n} = \frac{1/2}{1-1/2} \to S = 2$. Let T denote $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$. Then $\frac{T}{2} = \sum_{n=1}^{\infty} \frac{n^2}{2^{n+1}}$. Subtracting the two yields $\frac{T}{2} = \sum_{n=1}^{\infty} \frac{2n-1}{2^n}$. Dividing this by two results in $\frac{T}{4} = \sum_{n=1}^{\infty} \frac{2n-1}{2^{n+1}}$. Subtracting these two gives $\frac{T}{4} = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{2}{2^{n+1}} = \frac{3}{2} \to T = 6$. Hence, the sum is $S+T = 2+6 = \boxed{8}$.

3. B. Write out the sum as $\left(\frac{1}{n}\right)^n + \left(\frac{2}{n}\right)^n + \ldots + \left(\frac{n-2}{n}\right)^n + \left(\frac{n-1}{n}\right)^n + \left(\frac{n}{n}\right)^n$. Notice we can rewrite the sum, backwards, as $1^n + \left(1-\frac{1}{n}\right)^n + \left(1-\frac{2}{n}\right)^n + \left(1-\frac{3}{n}\right)^n + \ldots$. Taking the limit to infinity, we recognize each term after the first as a power of e: $1 + e^{-1} + e^{-2} + e^{-3} + \ldots = \frac{1}{1-e^{-1}} = \boxed{\frac{e}{e-1}}$

4. B. Since the harmonic series diverges, we know by definition that the sequence of partial sum diverges. Since the alternating harmonic series is conditionally convergent, by the Riemann Series Theorem, its terms can be arranged in a permutation so that the sum converges to any given value. D can be shown true by considering how many terms on each interval $\left[10^n, 10^{n+1}\right]$ are left over after terms are removed, finding an upper bound for the sum as a result, and by comparison test, it converges. While the harmonic series diverges, the ratio test is inconclusive.

5. C. If r is the proportion of rebound on each bounce, then the total distance the ball travels is $80 + 2(80)\left(\frac{r}{1-r}\right) = 320$. Solving, $r = \frac{3}{5}$, so it rebounds $\boxed{60}$% its previous height.

6. E. By Binomial Theorem, this is $(1.3+.7)^{11} = 2^{11} = \boxed{2048}$.

7. A. Notice we can rewrite this sum as $\sum_{n=1}^{\infty}\frac{1}{n^{2x^3+x^2-5x+3}}$. By the p-series test, this converges so long as $2x^3+x^2-5x+3>1$, or $2x^3+x^2-5x+2>0$. The roots of this are $1, \frac{1}{2},$ and -2. Testing intervals for positivity, we find that the positive area is $\boxed{\left(-2,\frac{1}{2}\right)\cup(1,\infty)}$.

8. A. Let the roots be $\frac{a}{r}, a,$ and ar. Then the product of the roots, $a^3=\frac{27}{8}\to a=\frac{3}{2}$. Taking the sum of the roots two at a time, $a^2\left(1+r+\frac{1}{r}\right)=\frac{78}{8}=\frac{39}{4}$. Letting $a=\frac{3}{2}$ and solving, we get $r=3,\frac{1}{3}$. So the roots are $\frac{3}{2},\frac{1}{2},$ and $\frac{9}{2}$, which has a sum of $\frac{13}{2}=\frac{p}{8}\to p=\boxed{52}$.

9. B. Writing this out, $\ln\left(\frac{2}{3}\right)+\ln\left(\frac{3}{4}\right)+\ln\left(\frac{4}{5}\right)+...+\ln\left(\frac{2013}{2014}\right)$. Using properties of logs, this is $\ln\left(\frac{2}{3}\cdot\frac{3}{4}\cdot\frac{4}{5}\cdot...\cdot\frac{2013}{2014}\right)$. Cancelling, we obtain $\ln\left(\frac{2}{2014}\right)=\boxed{\ln\left(\frac{1}{1007}\right)}$

10. A. Letting $n=x$ shows that $f(1)+f(2)+f(3)+...+f(x-1)+f(x)=x^2+2x$. Letting $n=x-1$ yields $f(1)+f(2)+f(3)+...+f(x-1)=(x-1)^2+2(x-1)$. Subtracting the two, $f(x)=x^2+2x-(x-1)^2-2(x-1)=2x+1$, so the sum of the coefficients is $\boxed{3}$.

11. A. The Rock's number is of the form $\left(\frac{1}{4}\right)^i$, and CM Punk's number is of the form $\left(\frac{1}{7}\right)^j$. What we need to find, then, is $\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\left(\frac{1}{4}\right)^i\left(\frac{1}{7}\right)^j$. Working from the inside out, we realize that $\left(\frac{1}{4}\right)^i$ can be factored out of $\sum_{j=1}^{\infty}\left(\frac{1}{4}\right)^i\left(\frac{1}{7}\right)^j$ to yield $\left(\sum_{i=1}^{\infty}\left(\frac{1}{4}\right)^i\right)\left(\sum_{j=1}^{\infty}\left(\frac{1}{7}\right)^j\right)$. Realizing these are both just infinite geometric series, we obtain $\sum_{i=1}^{\infty}\left(\frac{1}{4}\right)^i=\frac{1}{3}$ and $\sum_{j=1}^{\infty}\left(\frac{1}{7}\right)^j=\frac{1}{6}$, so the answer is $\boxed{\frac{1}{18}}$.

12. D. Note that $E_{n+1}-E_n=\frac{1}{2n+2}$, so $S_n=\frac{E_{n+1}-E_n}{E_nE_{n+1}}=\frac{1}{E_n}-\frac{1}{E_{n+1}}$. Then,
$\sum_{n=1}^{\infty}\left(\frac{1}{E_n}-\frac{1}{E_{n+1}}\right)=\frac{1}{E_1}=\boxed{2}$.

13. B. Note that we can split the sum up as $2\sum_{n=0}^{\infty}\frac{n^2}{n!}+\sum_{n=0}^{\infty}\frac{1}{n!}$. The second sum is clearly e. For the first sum, note that $n^2=n(n-1)+n$, so $\sum_{n=0}^{\infty}\frac{n^2}{n!}=\sum_{n=0}^{\infty}\frac{n(n-1)}{n!}+\sum_{n=0}^{\infty}\frac{n}{n!}$. Also recognize that $\sum_{n=0}^{\infty}\frac{n(n-1)}{n!}=\sum_{n=2}^{\infty}\frac{n(n-1)}{n!}=\sum_{n=2}^{\infty}\frac{1}{(n-2)!}=e$. Similarly, $\sum_{n=0}^{\infty}\frac{n}{n!}=\sum_{n=1}^{\infty}\frac{n}{n!}=\sum_{n=1}^{\infty}\frac{1}{(n-1)!}=e$. Thus, the entire sum is $2(2e)+e=\boxed{5e}$.

14. E. This is simply the tenth Fibonacci number. $\boxed{55}$

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15. A. Note that $49^{60}=(50-1)^{60}$, which from Binomial Theorem, is $\binom{60}{60}50^{60}-\binom{60}{59}50^{59}+...+\binom{60}{2}50^2-\binom{60}{1}50+\binom{60}{0}$. To find the last three digits, we need to find this sum modulo 1000. Notice that we only need to consider the last three terms, since any term which includes a multiple of 50^3 or higher will be a multiple of 1000. $\binom{60}{0}=1$, $50\binom{60}{1}=3000\equiv 0 \bmod 1000$, and $50^2\binom{60}{2}=30\cdot 50^2\cdot 59\equiv 0 \bmod 1000$. Hence, the last three digits are 0, 0, and 1, and the sum is $\boxed{1}$.

16. C. $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|=\lim_{n\to\infty}\left|\frac{(-1)^{n+1}(n+1)(x-2)^{n+1}3^n}{(-1)^n n(x-2)^n 3^{n+1}}\right|=\lim_{n\to\infty}\left|\frac{1}{3}(x-2)\frac{n+1}{n}\right|=\frac{1}{3}|x-2|$. For the series to converge this must be less than 1. $|x-2|<3\to\boxed{-1<x<5}$

17. D. By the P-Series Test, A converges. By the Alternating Series Test, B converges. By the Root Test, C converges. By the Divergence Test, D diverges.

18. A. Since $e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!}$, $e^{-2x^3}=\sum_{n=0}^{\infty}\frac{(-2x^3)^n}{n!}=\sum_{n=0}^{\infty}\frac{(-2)^n x^{3n}}{n!}$. Multiplying by x^5 yields

$$\boxed{\sum_{n=0}^{\infty}\frac{(-2)^n x^{3n+5}}{n!}}$$

19. B. Recall that $\sum_{n=1}^{\infty}x^n=\frac{x}{1-x}$. Dividing both sides by x and integrating with respect to x yields $\sum_{n=1}^{\infty}\frac{x^n}{n}=-\ln(1-x)$. Our desired sum occurs when we let $x=\frac{1}{3}$ to obtain $-\ln\left(\frac{2}{3}\right)=\boxed{\ln\left(\frac{3}{2}\right)}$.

20. D. The form provided hints at exponential growth. So we look for solutions to the recurrence of the form c^n for some constant c. Substituting, we get $c^n = -c^{n-1} + 6c^{n-2}$. Dividing through by c^{n-2} and getting everything on one side, we obtain our characteristic equation of $c^2 + c - 6 = 0$, which factoring yields solutions of $c = 2, -3$. Thus, $a = 2, b = -3$. Now, plugging in our initial conditions results in $\lambda_1 + \lambda_2 = 1$, and $2\lambda_1 - 3\lambda_2 = 3$. From the first equation, we now know $\lambda_1 + \lambda_2 + a + b = 1 + 2 - 3 = \boxed{0}$. If we wanted to solve the system, we'd obtain $\lambda_1 = \frac{6}{5}, \lambda_2 = -\frac{1}{5}$, so that $a_n = \frac{6}{5}(2)^n - \frac{1}{5}(-3)^n$. It's easily verifiable that this generates our recurrence.

21. A. Firstly, $\frac{1}{1-x} = 1 + x + x^2 + \ldots$ is the generating function for $(1,1,1,1,\ldots)$. Differentiating both sides, $\frac{1}{(1-x)^2}$ will thus result in the generating function for $(1,2,3,4,\ldots)$. Multiplying by x will result in $\frac{x}{(1-x)^2}$, the generating function for $(0,1,2,3,\ldots)$. Differentiating one last time will hence give coefficients that are exactly the positive perfect squares, and a final answer of $\boxed{\frac{1+x}{(1-x)^3}}$.

22. C. In the denominator, divide under the radical by n^2 to obtain $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{n\sqrt{1+\frac{4k^2}{n^2}}}$. We recognize this as a Riemann Sum. To convert to an integral, let $\frac{k}{n} = x$ so that $\frac{1}{n} = dx$. The integral, then, is $\int_0^1 \frac{1}{\sqrt{1+4x^2}}\,dx$. We can evaluate this by letting $x = \frac{1}{2}\tan\theta$ so that $dx = \frac{1}{2}\sec^2\theta d\theta$. The integral would hence become $\frac{1}{2}\int_0^{\tan^{-1}(2)} \sec\theta d\theta$. Recall that $\int \sec\theta d\theta = \ln\left|\sec\theta + \tan\theta\right|$. Thus, we obtain $\frac{1}{2}\ln\left(\sec\left(\tan^{-1}(2)\right) + 2\right) = \boxed{\frac{1}{2}\ln\left(2+\sqrt{5}\right)}$.

23. C. $f(x) = f(a) + f'(a)(x-a) + \frac{1}{2} f''(a)(x-a)^2$, given $a = 4$ and $f(x) = \ln(x)$. So $f'(x) = \frac{1}{x}$ and $f''(x) = -\frac{1}{x^2}$. Plugging in the values gives the desired result.

24. C Note that $\sum_{n=0}^{\infty}(-1)^n \tan^{2n}(\theta)=1-\tan^2\theta+\tan^4\theta-...=\frac{1}{1+\tan^2\theta}=\frac{1}{\sec^2\theta}=\cos^2\theta$. Then, $\cos^2\theta=1-\sin^2\theta=\sin^2\theta-3\sin\theta+2\to 2\sin^2\theta-3\sin\theta+1=0$. Factoring, $(2\sin\theta-1)(\sin\theta-1)=0\to\sin\theta=\frac{1}{2},\sin\theta=1$. On $[0,2\pi]$, the solutions are $\frac{\pi}{2},\frac{\pi}{6}$, and $\frac{5\pi}{6}$ for a total of $\boxed{\frac{3\pi}{2}}$.

25. B. Using linearity, we can split this up as $\sum_{i=1}^{12}\left(\sum_{j=1}^{i}i+\sum_{j=1}^{i}j\right)=\sum_{i=1}^{12}\sum_{j=1}^{i}i+\sum_{i=1}^{12}\sum_{j=1}^{i}j$.

$\sum_{i=1}^{12}\sum_{j=1}^{i}i=\sum_{i=1}^{12}i^2=\frac{12(13)(25)}{6}=650$ and $\sum_{i=1}^{12}\sum_{j=1}^{i}j=\sum_{i=1}^{12}\frac{i(i+1)}{2}=\frac{1}{2}\sum_{i=1}^{12}i^2+\frac{1}{2}\sum_{i=1}^{12}i=325+39=364$.

Hence, $650+364=\boxed{1014}$.

26. D. Let S denote the original sum, and note the symmetry: $\binom{11}{i}=\binom{11}{11-i}$. Adding S to itself, we obtain $2S=11\left(\binom{11}{1}+\binom{11}{2}+\binom{11}{3}+...+\binom{11}{10}\right)+11\binom{11}{11}\to S=\frac{11\left(2^{11}-2\right)}{2}+11=\boxed{11264}$

27. A. Note that $|\sin(n)|\le 1$, and so $\left|\frac{\sin(n)}{n!}\right|\le\frac{1}{n!}$. By Comparison Test, this series is absolutely convergent.

28. E. When the product is expanded, we end up with a product of cosines over a product of sines. Due to the cofunction identity $\cos\left(\frac{\pi}{2}-x\right)=\sin(x)$, the numerator and denominator have precisely the same entries, so the product is $\boxed{1}$.

29. E. None of the answers must necessarily be true.

30. C. Recall that $\sum_{n=0}^{\infty}\frac{x^{2n}(-1)^n}{(2n)!}=\cos(x)$. So, $\sum_{n=0}^{\infty}\frac{3^n(-1)^n}{(2n)!}=\sum_{n=0}^{\infty}\frac{\left(\sqrt{3}\right)^{2n}(-1)^n}{(2n)!}=\boxed{\cos\left(\sqrt{3}\right)}$.