Answers:

0. 60
1. 3
2. $1/2$ or 0.5
3. 6
4. 144
5. -22
6. 4.9
7. $\frac{13\sqrt{26}}{3}\pi$
8. 4
9. $y=13$ and $y=-5$
10. $\tan\left(\theta/2\right)$
11. $2\sqrt{3}$
12. 1002

Solutions:

0. Multiplying the first equation by 7 and the second equation by 8, then adding the two resulting equations together yields $191x = 2865 \Rightarrow x = 15$. Plugging this back in and solving for *y* yields $y = 12$. The least common multiple of 15 and 12 is 60.

1. Using the method of finite differences,

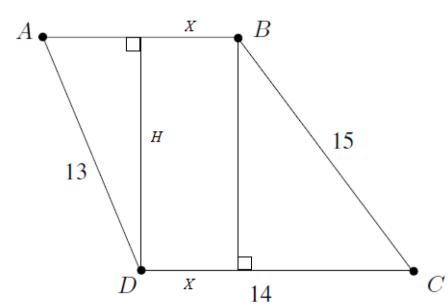
$$\begin{array}{ccccccccccc}
-5 & & 0 & & 9 & & 16 & & 15 & & 0 \\
 & 5 & & 9 & & 7 & & -1 & & -15 & \\
 & & 4 & & -2 & & -8 & & -14 & & \\
 & & & -6 & & -6 & & -6 & & &
\end{array}$$

Since we had to go down three levels, there is a cubic polynomial satisfying the given conditions but no quadratic, linear, or constant polynomial; thus, $n = 3$.

2. By plugging in the given points, we get that $5 = a \cdot 6^b$ and $1 = a \cdot 2^b$. Dividing the corresponding parts of these equations by each other yields $5 = 3^b \Rightarrow b = \log_3 5$.
Plugging this back into the first equation, $5 = a \cdot 6^{\log_3 5} = a \cdot 5^{\log_3 6} \Rightarrow a = 5^{1-\log_3 6} = 5^{\log_3\left(1/2\right)}$.
Taking $\log_5$ of both sides of this equation yields $\log_5 a = \log_3\left(1/2\right) \Rightarrow c = 1/2$.

3. Let *a* be the length of the apothem. Then $2 \cdot \dfrac{a}{\sqrt{3}} = \dfrac{2a}{\sqrt{3}}$ is the length of a side of the hexagon, meaning that $72\sqrt{3} = \dfrac{1}{2} a \cdot \dfrac{12a}{\sqrt{3}} = 2\sqrt{3}a^2 \Rightarrow a^2 = 36 \Rightarrow a = 6$ (since $a > 0$).

4. Draw altitudes from *D* and *B* to the opposite sides; label sides as in the diagram. Then $H^2 + \left(10 - X\right)^2 = 169$ and $H^2 + \left(14 - X\right)^2 = 225$. Subtracting the first equation from the second yields $196 - 28X + X^2 - 100 + 20X - X^2 = 56$ $\Rightarrow 40 = 8x \Rightarrow x = 5 \Rightarrow H = 12$. Therefore, the enclosed area is $\dfrac{1}{2} \cdot 10 \cdot 12 + \dfrac{1}{2} \cdot 14 \cdot 12 = 144$.



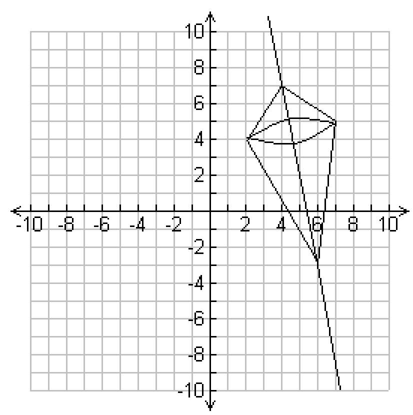
5. Expanding by the first row, $\begin{vmatrix} 0 & 1 & 3 & -1 \\ -2 & -1 & 0 & -3 \\ 1 & -1 & 1 & 0 \\ 3 & -1 & -2 & 2 \end{vmatrix} = -\begin{vmatrix} -2 & 0 & -3 \\ 1 & 1 & 0 \\ 3 & -2 & 2 \end{vmatrix} + 3\begin{vmatrix} -2 & -1 & -3 \\ 1 & -1 & 0 \\ 3 & -1 & 2 \end{vmatrix}$

$+\begin{vmatrix} -2 & -1 & 0 \\ 1 & -1 & 1 \\ 3 & -1 & -2 \end{vmatrix} = -(-4+0+6+9-0-0)+3(4+0+3-9-0+2)+(-4-3+0-0-2-2)$

$= -11+3(0)-11 = -22$.

6. The number of ways of being dealt three cards of three distinct ranks is $\binom{13}{3}\binom{4}{1}^3$ $=18304$. The number of ways of being dealt one pair with another card of a different rank than the pair is $\binom{13}{1}\binom{4}{2}\binom{12}{1}\binom{4}{1} = 3744$. Therefore, Pablito is $\dfrac{18304}{3744} = 4.888...$ more times likely to be dealt three cards of three distinct ranks than to be dealt one pair with another card of a different rank than the pair. This answer rounded to the nearest tenth is 4.9.

7. The line actually contains the first and third points, so when the triangle is revolved, it will consist of two cones with a shared base. If we factor out what is common, then the sum of the volumes of the two cones would be $\frac{1}{3}\pi r^2(h_1+h_2)$, where h_1 and h_2 are the altitudes of the two cones. However, as seen in the diagram, h_1+h_2 would just be the distance between the two points on the line, and r would be the distance from the second point listed to the line. Therefore, h_1+h_2 $=\sqrt{(4-6)^2+(7+3)^2}=\sqrt{104}=2\sqrt{26}$ and $r=\dfrac{|5\cdot 7+5-27|}{\sqrt{5^2+1^2}}=\dfrac{13}{\sqrt{26}}=\dfrac{\sqrt{26}}{2}$. This makes the volume $\dfrac{1}{3}\pi\left(\dfrac{\sqrt{26}}{2}\right)^2\left(2\sqrt{26}\right)=\dfrac{13\sqrt{26}}{3}\pi$.



8. If $x<1$, the equation becomes $8=1-x+2-x+3-x=6-3x \Rightarrow 3x=-2 \Rightarrow x=-\frac{2}{3}$, which is less than 1. If $1<x<2$, the equation becomes $8=x-1+2-x+3-x=4-x$

$\Rightarrow x = -4$, which is not between 1 and 2. If $2 < x < 3$, the equation becomes $8 = x - 1 + x - 2 + 3 - x = x \Rightarrow x = 8$, which is not between 2 and 3. If $x > 3$, the equation becomes $8 = x - 1 + x - 2 + x - 3 = 3x - 6 \Rightarrow 3x = 14 \Rightarrow x = {}^{14}\!/_{3}$, which is greater than 3. Therefore, the two solutions are $-{}^{2}\!/_{3}$ and ${}^{14}\!/_{3}$, and their sum is 4.

9. The directrices are a distance ${}^{a^2}\!/_{c}$ from the center, running perpendicular to the major axis. $a^2 = 36$ and $c^2 = 36 - 20 = 16 \Rightarrow c = 4$; therefore, the directrices are a distance ${}^{36}\!/_{4} = 9$ from the center. Further, since the larger number is underneath the *y*-term in the equation of the ellipse, the major axis is vertical, meaning the directrices are horizontal. Thus the two equations are $y = 4 \pm 9$; i.e., $y = 13$ and $y = -5$.

10. $\sin\theta - \sin\theta\cos\theta + \ldots + \sin\theta(-\cos\theta)^{n-1} + \ldots = \dfrac{\sin\theta}{1-(-\cos\theta)} = \dfrac{\sin\theta}{1+\cos\theta} = \tan\left({}^{\theta}\!/_{2}\right)$

11. The volume of a right hexagonal prism is $\dfrac{3\sqrt{3}}{2}a^2h$, where *a* and *h* are the edge length of the base and a height, respectively. Since all edges are the same length, $108 = \dfrac{3\sqrt{3}}{2}a^3 \Rightarrow a^3 = 24\sqrt{3} \Rightarrow a = 2\sqrt{3}$.

12. If written in base-12, each 0 at the end would required two 2s and a 3 as factors. The number of 3s in the factorization of $2015!$ is $\sum_{n=1}^{\infty}\left\lfloor \dfrac{2015}{3^n} \right\rfloor = 671 + 223 + 74 + 24 + 8 + 2 = 1002$, and the number of 2s in the factorization of $2015!$ is $\sum_{n=1}^{\infty}\left\lfloor \dfrac{2015}{2^n} \right\rfloor = 1007 + 503 + 251 + 125 + 62 + 31 + 15 + 7 + 3 + 1 = 2005$. Since one 3 is needed for every two 2s, we will run out of 3s first when pairing these factors, so there are 1002 consecutive zeros at the end of $2015!$ when written in base-12.