Theta Ciphering 2016 Mu Alpha Theta National Convention

Answers

0. $280x^3$
1. $-\frac{23}{7}$
2. 34
3. $\frac{3}{2}$
4. 8
5. $2\sqrt{13}$
6. 337
7. 133200_4 or 133200
8. 65
9. 15 ft
10. 2 or (2, 0)
11. $162\sqrt{3}-81\pi$
12. 18
13. $-\frac{4}{3}$
14. 151,200

Solutions:

0. $\binom{7}{3}(2x)^3(-1)^4 = 280x^3$

1. $\left(\frac{2}{3}\right)^{2(2x+7)} = \left(\frac{2}{3}\right)^{-3(x+3)} \Rightarrow 4x+14=-3x-9 \Rightarrow 7x=-23 \Rightarrow x=-\frac{23}{7}$

2. $7 = x^2-y^2 = (x-y)(x+y) = (x-y)(-1) \Rightarrow x-y=-7$. The solution to the system of equations $x+y=-1$, $x-y=-7$ is $(-4,3)$, so $5x+6y-3xy = 5\cdot -4+6\cdot 3-3\cdot -3\cdot 4 = -20+18+36=34$.

3. Let $u=\sqrt{2x-1}$. Then $\sqrt{\frac{3(u^2+1)}{2}}+u=\frac{5}{u} \Rightarrow \sqrt{\frac{3(u^2+1)}{2}} = \frac{5-u^2}{u} \Rightarrow \frac{3u^2+3}{2} = \frac{u^4-10u^2+25}{u^2}$ $\Rightarrow 3u^4+3u^2 = 2u^4-20u^2+50 \Rightarrow 0 = u^4+23u^2-50 = (u^2+25)(u^2-2)$, and since $u \geq 0$, $u=\sqrt{2}$. Therefore, $2x-1=2 \Rightarrow x=\frac{3}{2}$.

4. Let $a=\log 4$ and $b=\log 25$. Then we are looking for $3ab^2+b^3+a^3+3a^2b=(a+b)^3$, and since $a+b=\log 4+\log 25=\log 100=2$, $(a+b)^3=8$.

5. $y=\frac{1}{2}x^2-6x+15=\frac{1}{2}(x-6)^2-3$, so the vertex is $(6,-3)$. $x^2-4x+y^2-6y=5 \Rightarrow (x-2)^2+(y-3)^2$ $=18$, so the center is $(2,3)$. The distance between these points is $\sqrt{(2-6)^2+(-3-3)^2}=\sqrt{16+36}$ $=\sqrt{52}=2\sqrt{13}$.

6. By the given information, $\sqrt{xy}=12 \Rightarrow xy=144$ and $\frac{x+y}{2}=12.5 \Rightarrow x+y=25$. Since $(x+y)^2$ $=x^2+y^2+2xy$, $625=25^2=x^2+y^2+2\cdot 144=x^2+y^2+288 \Rightarrow x^2+y^2=337$.

7. Since $2016=1\cdot 4^5+3\cdot 4^4+3\cdot 4^3+2\cdot 4^2+0\cdot 4^1+0\cdot 4^0$, $2016=133200_4$.

8. Todd travels 30 miles/hour for 0.8 hours, or 24 miles. Calvin travels 24 miles per hour for 5/12 of an hour, or 10 miles. Since they traveled in perpendicular directions, they are 26 miles apart. Since they are traveling toward each other, each at 12 miles/hour, they are closing the gap at 24 miles/hour. Therefore, it will take 26/24 = 13/12 hours, or 65 minutes.

9. No matter how far apart the poles are, the wires will cross each other at a height of $\frac{a\cdot b}{a+b}$ units above ground, where *a* and *b* are the heights of the two poles in the same units. Therefore, the intersection of the wires occurs at a height of $\frac{24\cdot 40}{24+40}=15$ feet.

10. The line $5x-2y=6$ has slope $\frac{5}{2}$, so a perpendicular line has slope $-\frac{2}{5}$. The midpoint of the line segment whose endpoints are $(-5,-3)$ and $(-1,7)$ is $\left(\frac{-5+(-1)}{2},\frac{-3+7}{2}\right)=(-3,2)$, so the line we are looking for has equation $y-2=-\frac{2}{5}(x+3)$. The x-intercept has y-coordinate 0, so $-2=-\frac{2}{5}(x+3)$ $\Rightarrow 5=x+3 \Rightarrow x=2$.

11. The radius of the circle is an apothem of the hexagon, so a side length of the hexagon is $\frac{9}{\sqrt{3}}\cdot 2=6\sqrt{3}$. Therefore, the sought area is $\frac{1}{2}\cdot 9\cdot\left(6\cdot 6\sqrt{3}\right)-\pi\cdot 9^2=162\sqrt{3}-81\pi$.

12. Let R and r be the lengths of the circumscribed and inscribed circles, respectively. Then $R=2r+8$, and therefore, $12=\sqrt{(2r+8)((2r+8)-2r)}=4\sqrt{r+4}\Rightarrow r=5\Rightarrow R=18$.

13. $6+0-4-0-0-4x=0-6x-2-x-0-0\Rightarrow 2-4x=-7x-2\Rightarrow 4=-3x\Rightarrow x=-\frac{4}{3}$

14. There are 2 Os, 2 Ks, and 3 Es, so the number of distinct permutations is $\frac{10!}{2!2!3!}=151,200$.