1. Given the diagram with circle P and tangent $\overline{AB}$; if $AB = 2x+4$ and $BC = x+6$, and the radius of the circle is $x$, then what is the perimeter of triangle $ABP$?

A. 10  B. 30  C. 60  D. 100  E. NOTA

2. A convex polygon has exterior angles whose measures in degrees are distinct consecutive positive odd integers. What is the greatest number of sides this polygon can have?

A. 13  B. 15  C. 17  D. 19  E. NOTA

3. Given circle $O$ with enclosed area $36\pi$, and chord $\overline{JK}$ of length $6\sqrt{2}$, and a randomly selected point $L$ on circle $O$. What is the probability that this randomly selected point will make $\triangle JKL$ obtuse?

A. $\frac{1}{2}$  B. $\frac{1}{4}$  C. $\frac{2}{3}$  D. $\frac{3}{4}$  E. NOTA

4. Consider all possible regular $n$-gons which can tessellate a plane using infinitely many copies of one particular type of $n$-gon. That is to say, you can completely cover the plane without overlapping. For each value of $n$, take the measure of one interior angle for this regular $n$-gon. What is the sum, in degrees, of all possible of these interior angles, one for each $n$-gon?

A. 150  B. 270  C. 360  D. 440  E. NOTA

5. A regular polygon is inscribed in a circle of radius 8. Each interior angle of this polygon has measure $165^\circ$. If the vertices are labeled with consecutive letters of the English alphabet, starting with the letter A, and proceeding in a clockwise fashion, then which vertex is at the other end of a diameter containing vertex $N$?

A. A  B. B  C. C  D. D  E. NOTA

6. A certain circle has an inscribed angle $\angle BDC$ whose measure is $50^\circ$. If point A is outside the circle so that $\overline{AB}$ and $\overline{AC}$ are tangent to circle P, then what is $m\angle BAC$, in degrees?

A. 120  B. 100  C. 80  D. 60  E. NOTA

7. The sandbox on Polly's Polygonal Playground is in the shape of a dodecagon. Paul and Polly play a game where they start by standing on the same vertex, and every 10 seconds they rotate counterclockwise, with Paul going to the 5th vertex and Polly going to the 7th vertex from where each of them was previously. The game is over when they are both standing on the vertex where the game began at exactly the same time. During this game, how many more complete revolutions will Polly make around this sandbox than Paul?

A. 2  B. 4  C. 6  D. 24  E. NOTA
8. There are 2015 congruent toothpicks in a bin. You pull them out and began to construct one of each type of regular polygon, starting with a triangle, and each successive polygon having one additional side, leaving each shape in-tact and never using one toothpick for multiple shapes. What is the measure of one exterior angle of the last polygon you could completely construct?

A. \(\frac{40}{7}\)  
B. \(\frac{45}{8}\)  
C. \(\frac{180}{31}\)  
D. \(\frac{9}{2}\)  
E. NOTA

9. What is the area enclosed by a circle passing through the points (-3, -2), (5, -2), and (-9, 4)?

A. \(16\pi\)  
B. \(64\pi\)  
C. \(128\pi\)  
D. \(116\pi\)  
E. NOTA

10. A square is inscribed in a circle. Then both of these shapes are revolved around one of the diagonals of the square. What is the ratio of the volume of the solid that the square “sweeps out” to the volume of the solid that the circle “sweeps out”?

A. 4:5  
B. 3:4  
C. 2:3  
D. 1:2  
E. NOTA

11. There exist polygons such that a polygon with X sides has Y letters in its spelled-out English name, and a polygon with Y sides has X letters in its spelled-out English name. What is the smallest possible value for X + Y?

A. 7  
B. 11  
C. 15  
D. 19  
E. NOTA

12. Carl Friedrich Gauss was a mathematician born in the late eighteenth century who, at the age of 19, gave the world its first progress in regular polygon construction in about 2000 years. His proof, which gave him notoriety as one of the best mathematicians of the day, involved the construction of a polygon with how many sides (using only a straightedge and compass)?

A. 9  
B. 13  
C. 17  
D. 21  
E. NOTA

13. Find the measure, in degrees, of one interior angle of a regular decagon.

A. 36  
B. 30  
C. 150  
D. 144  
E. NOTA

14. In a circle, two perpendicular chords, namely \(\overline{AB}\) and \(\overline{CD}\), are drawn intersecting at point E. If \(DE = 8\), \(CE = 18\), and \(AE:BE = 1:4\), then what is the length of the radius of the circle?

A. \(5\sqrt{10}\)  
B. \(3\sqrt{41}\)  
C. \(\sqrt{269}\)  
D. \(17\)  
E. NOTA

15. Jane and Jill both have polygons. Jane’s polygon has 3 more sides and exactly 3 times the number of diagonals when compared with Jill’s polygon. What is the sum of the numbers of sides of their two polygons?

A. 9  
B. 15  
C. 21  
D. 27  
E. NOTA

16. A regular octagon is inscribed in a circle of radius 1. What is the length of a side of the octagon?

A. \(2 - \sqrt{3}\)  
B. \(2 - \sqrt{2}\)  
C. \(\sqrt{2} - \sqrt{3}\)  
D. \(\sqrt{2} - \sqrt{2}\)  
E. NOTA

17. Two externally tangent circles have radii 8 and 18. What is the length of a common external tangent (distance between tangency points) for these two circles? (Round to the nearest integer.)

A. 24  
B. 23  
C. 22  
D. 21  
E. NOTA
18. Euclid’s Geometry textbook has a problem where he is to find the measure of an interior angle of a regular polygon rounded to the nearest degree. He quickly gets the answer of 180°, which surprisingly enough is correct. What is the sum of the digits in the smallest possible number of sides this polygon could have?
   A. 9  B. 10  C. 11  D. 17  E. NOTA

19. The star shown can be inscribed in a circle.
   Find $m\angle A + m\angle B + m\angle C + m\angle D + m\angle E$, in degrees.
   A. 180  B. 240  C. 300  D. 360  E. NOTA

20. A certain quadrilateral can be cut into 4 congruent triangles, all of which have a common vertex. What is the most general name which can correctly be given to this quadrilateral based only on this information?
   A. Parallelogram  B. Rhombus  C. Rectangle  D. Square  E. NOTA

21. Ten points are equally spaced around the perimeter of a semicircle (including the diameter), with 4 of the 10 points lying on the diameter. What is the maximum number of triangles that can be formed using these 10 points?
   A. 112  B. 116  C. 120  D. 124  E. NOTA

22. Given regular octagon ABCDEFGH, what is $m\angle AFD$ (in degrees)?
   A. 90  B. 72  C. 67.5  D. 62.25  E. NOTA

23. In a certain polygon, $d$ is the number of diagonals that can be drawn from each vertex. Which expression represents the number of total diagonals which can be drawn in this polygon?
   A. $\frac{1}{2}d^2 + \frac{3}{2}d$  B. $\frac{1}{2}d^2 - \frac{3}{2}d$  C. $\frac{1}{2}d^2 + \frac{1}{2}d - 1$  D. $\frac{1}{2}d^2 - \frac{1}{2}d - 1$  E. NOTA

24. The perimeter of a rhombus is 100, and one diagonal has length 14. What is the area enclosed by this rhombus?
   A. 48  B. 168  C. 336  D. 672  E. NOTA

25. At a fair (which isn’t very fair at all), a game is played where you throw a dart at the dart board below. If you land inside the square, but not inside any circle, you win. What is the probability you will win, given that you hit the dart board every time, but you are not skilled at all beyond that? (Each circle is tangent to 2 other circles and to 2 sides of the board.)
   A. $\frac{\pi}{4}$  B. $\frac{\pi}{2}$  C. $\frac{4-\pi}{4}$  D. $\frac{2}{\pi}$  E. NOTA

26. A quadrilateral is inscribed in a circle. The measures of the arcs intercepted by 3 of the angles of the quadrilateral are 250°, 130°, and 110°. What is the measure of the arc intercepted by the other angle?
   A. 230°  B. 210°  C. 190°  D. Cannot be determined  E. NOTA
27. Given the diagram with circle G and chords $AB$ and $CD$, if $AB = 8$, $GE = 3$, and $GF = 4$, then what is the length of $CD$?

A. 8  B. 7  C. 6  D. 5  E. NOTA

28. The area enclosed by a certain octagon is 432, and its perimeter is 108. The perimeter of a similar polygon is 27. What is the area enclosed by this similar polygon?

A. 27  B. 54  C. 81  D. 108  E. NOTA

29. The sum of all but one interior angle of a convex polygon is $1524^\circ$. What is the product of the digits in the measure of the remaining interior angle, in degrees?

A. 27  B. 54  C. 81  D. 108  E. NOTA

30. Let a regular $n$-pointed star be a figure formed entirely of congruent diagonals of a regular $n$-gon with radius 1. There is one distinct way to draw a regular 5-pointed star. If the 5 points in clockwise fashion are A, B, C, D, and E, then you could form the star by connections A to C, C to E, E to B, B to D, then D to A. Any other method of connecting the points would be a reflection or rotation of this method, which creates a congruent star. There are two distinct ways to draw a regular 7-pointed star (connecting the points in the order A, C, E, G, B, D, F, A or in the order A, D, G, C, F, B, E, A). How many distinct ways are there to draw a regular 13-pointed star?

A. 4  B. 5  C. 6  D. 7  E. NOTA