Answers:

1. D
2. A
3. C
4. A
5. E
6. B
7. B
8. B
9. D
10. C
11. C
12. D
13. D
14. A
15. C
16. C
17. A
18. B
19. B
20. B
21. C
22. D
23. B
24. B
25. C
26. D
27. A
28. C
29. C
30. E

Solutions:

1. The adjacent exterior angle is 1° since the polygon is regular the number of sides is found by 360°/1° = 360.

2. Sum of interior angles $= (n-2)180;\ \ 1800 = (n-2)180;\ \ 10 = n-2;\ n = 12$

3. Sum of exterior angles = 360°; 50 + 90 + 100 + 2x = 360°; 2x = 120°; x = 60°

4. Isosceles triangle on each side of the pentagon. Each base angle of the triangle is 72° since it is the supplement of the 108° interior angle of the regular pentagon. Since the base angles sum is 144°, the vertex angle of the triangle is 36° which would be one of the five acute interior angles of this star. Sum = 5(36°) = 180°

5. The only midpoint of a side that is equidistant from the vertices of a triangle is the midpoint of the hypotenuse of a right triangle. Therefore, side #1 is the hypotenuse and sides #2 and #3 are the legs of a right triangle and the distance the midpoint is from the right angle vertex is the length of half the hypotenuse. Use the Pythagorean theorem to find the hypotenuse: $\#1 = \sqrt{25+36};\ \ \#1 = \sqrt{61};$ distance to vertex = $\sqrt{61}/2$

6. Counter examples for choice: A) – square and rhombus; C) – square and rhombus; D) – square and rectangle. Choice B) will result in congruent corresponding sides and angles and therefore congruent parallelograms.

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7. See the sketch to the right.

$$2^2 + d^2 = 12^2$$
$$4 + d^2 = 144$$
$$d^2 = 140$$
$$d = 2\sqrt{35}$$

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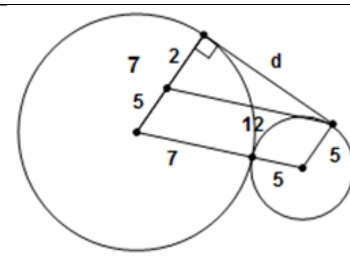

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8. See the sketch to the right

$$(40-x) + (9-x) = 41$$
$$-2x = -8$$
$$x = 4$$

Since x is the radius of the circle, the circumference of the circle is 8π

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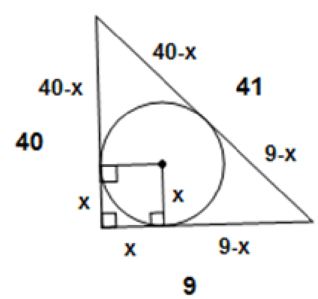

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9. Since the sides and vertices of the polygon are equidistant from the center of a circle, the polygon is regular and the shortest distance from the center to a side is the apothem.

$$360°/120° = 3\ sides;\qquad 60in/3 = 20\ in\ sides$$
$$Central\ angle\ measure = 360°/3 = 120°$$

This apothem is the short leg of 30-60-90 triangle whose long leg is 10 in. $Apothem = 10/\sqrt{3} = 10\sqrt{3}/3$

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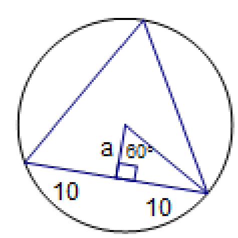

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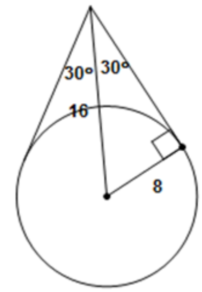
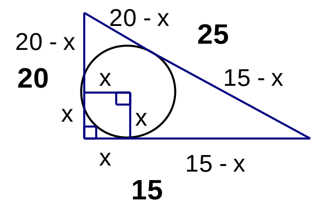
10. See the sketch at the right.

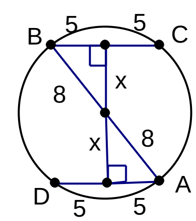
A 30-60-90 triangle is formed by connecting the external point with the center of the circle and drawing the radius to a point of tangency. The radius is now the short leg of the triangle and is half the length of the hypotenuse (which is the distance the external point is from the center of the circle).
Hypotenuse = 2(8) = 16



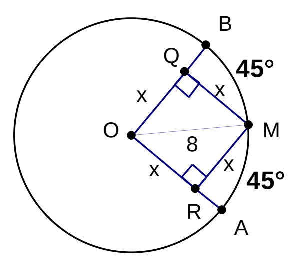
11. Since the measurements fit the Pythagorean Theorem, the triangle is a right triangle. The sketch at the right shows the inscribed circular hole to be inscribed in the triangle. Using the hypotenuse segment will yield the equation 20 – x + 15 – x = 25 and the solution for the radius "x"
- 2x = - 10; x = 5



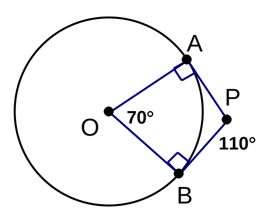
12. Segments $\overline{AD}$ and $\overline{BC}$ are drawn from opposite ends of the same diameter and since the segments are congruent their arcs are congruent and the remaining arc of each semi-circle are congruent $\left(\widehat{AC} \cong \widehat{BD}\right)$ and the two inscribed angles that intercept those arcs are congruent $(\measuredangle B \cong \measuredangle A)$. Since those angles are alternate interior angles for $\overline{AD}$ and $\overline{BC}$ those segments are parallel and the distance between $\overline{AD}$ and $\overline{BC}$ is 2x; $x^2 + 25 = 64$; $x^2 = 39$; $x = \sqrt{39}$; $2x = 2\sqrt{39}$



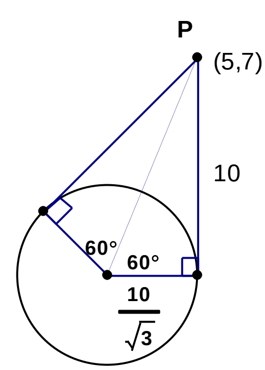
13. If arc AB is 90° the two arcs formed by its midpoint are 45° and as shown in the sketch at the right OQMR is a quadrilateral formed by two 45-45-90 triangles that share hypotenuse/radius OM. Since OM = 8 each leg (x) will be $4\sqrt{2}$ and perimeter (4x) is $16\sqrt{2}$



14. See sketch at right. Tangents from 90° angles with radii; central angle is 70°; sum of angles of a quadrilateral is 360°.
$m\measuredangle APB = 360 - 90 - 90 - 70 = 110$

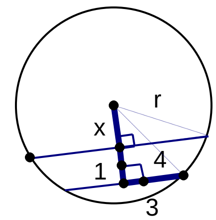


15. If one of the radii drawn to a point of tangency is horizontal the slope of the line can be determined by finding the length of the radius. See the sketch at the right. $m = 10/\left(10/\sqrt{3}\right) = \sqrt{3}$
Equation in point slope form: $y - 7 = \sqrt{3}(x - 5)$



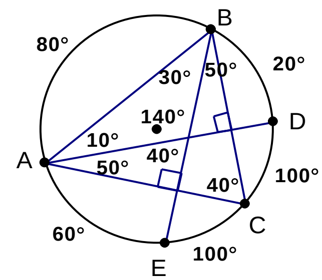
16. Since neither chord is the diameter nor are they on the same side of the center, we can write two equations that are both equal to the radius squared.

$$r^2 = x^2 + 16 = x^2 + 2x + 1 + 9$$
$$x^2 + 16 = x^2 + 2x + 1 + 9$$
$$6 = 2x$$
$$x = 3 \qquad r^2 = x^2 + 16 = 9 + 16 =$$
$$25; \quad r = 5$$



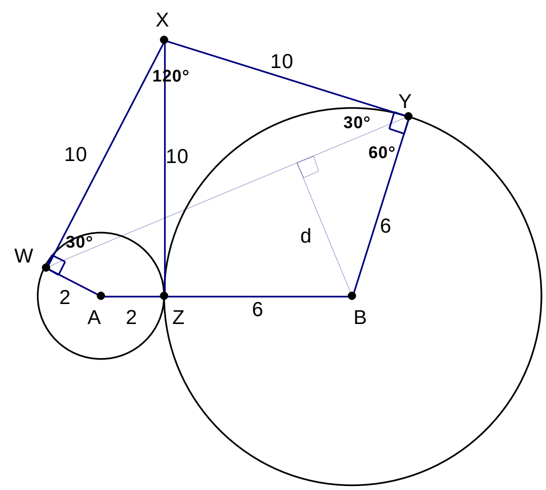
17. Since the triangle is acute none of its sides can be a diameter. See the sketch at the right. If angle BAD is 10 then arc BD is 20. If angle C is 40 then arc AB is 80, angle EBC is 50, arc EC is 100 and the acute angles formed by chords AD and BE is 40.

$$40 = .5(20 + AE)$$
$$80 = 20 + AE$$
$$60 = AE$$



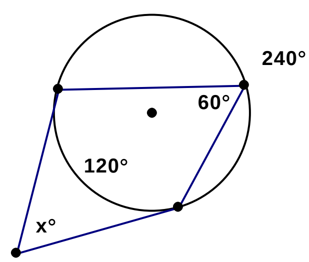
18. See sketch at the right. Since triangle XWY is isosceles with vertex angle of 120 its base angles are 30. The adjacent angle at Y must be 60 making the distance (d) we want the long leg of a 30-60-90 triangle whose hypotenuse is 6.

$$d = 3\sqrt{3}$$

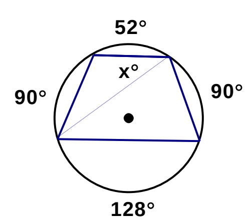


19. The arc of the inscribed 60° angle is 120° and the rest of the circle would be 240°. The angle formed by the tangents can be found by taking half the difference of its arcs.

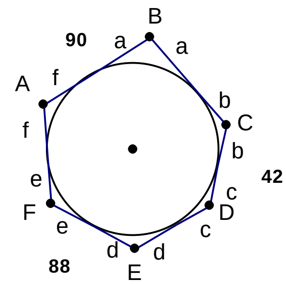
$$x° = .5(240° - 120°) = 60°$$



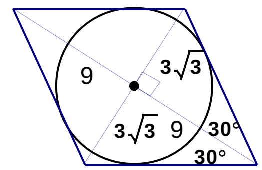
20. This must be an isosceles trapezoid since the arcs subtended by parallel lines will be congruent. The measure of each of those arcs will be $\frac{(360° - 52° - 128°)}{2} = \frac{180°}{2} = 90°$. The angle formed by a diagonal of the trapezoid and the shorter base intercepts one of the 90° arcs and therefore measures 45°



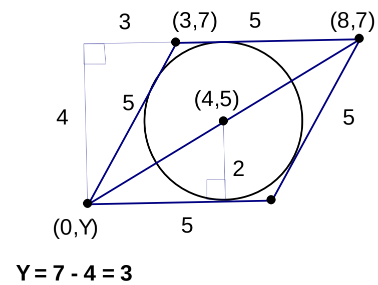
21. Perimeter equals 2a+2b+2c+2d+2e+2f
a + f =90; b + c= 42; d + e = 88 so a + b + c + d + e + f=220 and the perimeter is 440



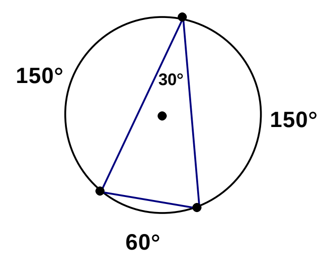
22. The parallelogram is a rhombus since tangents to the same circle are congruent. The diagonals from 30-60-right triangles with long leg of 9 and a short leg of $3\sqrt{3}$. The area of the parallelogram = $.5(18)(6\sqrt{3}) = 54\sqrt{3}$



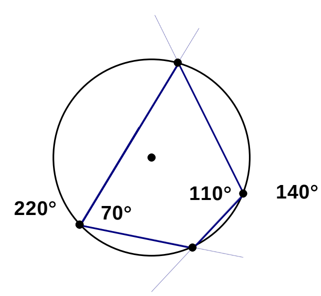
23. The parallelogram must be a rhombus (see #22). The points (3,7) and (8,7) give a side length of 5. In the dotted right triangle and horizontal leg equals the difference of the "x" values 0 & 3 and the triangle is a 3-4-5 triple. Therefore y = 3 and the midpoint of the long diagonal is (4,5) and the radius of the circle is 2.
Equation of circle: $(x-4)^2 + (y-5)^2 = 2^2$



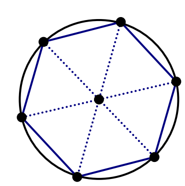
24. See sketch



25. Opposite angles of a quadrilateral inscribed in a circle are supplementary. 180 -70 = 110



26. $C = \pi d = 50\pi; d = 50; r = 25$ radius of circle is the radius of the regular hexagon and is the side length of the regular hexagon. Since a regular hexagon is formed by 6 equilateral triangles $A = 6\left(\frac{25^2\sqrt{3}}{4}\right) = \frac{1875\sqrt{3}}{2}$

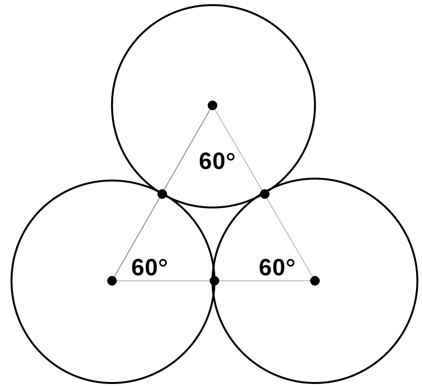


27. Since the circles are congruent the triangle is equilateral with a side length equal to a diameter. Desired area is 3 circles minus area of a semicircle.

Area of equilateral triangle:

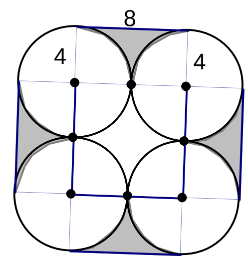
$d^2\sqrt{3}/_4 = 4\sqrt{3}\,;\ d^2 = 16;\ d = 4; r = 2.$

Desired area: $3(4\pi) - .5(4\pi) = 10\pi$



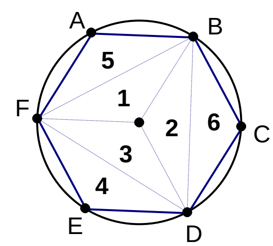
28. One shaded area = rectangle minus semicircle = $(8)(4) - .5(16\pi) = 32 - 8\pi$

Total shaded area = 4 times one shaded area = $4[32 - (8\pi)] = 128 - 32\pi$



29. As shown in the sketch, the regular hexagon and equilateral triangle can be conveniently drawn. By drawing the dotted segments, 6 congruent triangles are formed.

$$\Delta BDF : ABCDEF = \frac{\Delta's1,2,3}{\Delta's1,2,3,4,5,6} = \frac{3\Delta's}{6\,\Delta's} = \frac{1}{2}$$



30. $\frac{40}{x} = \frac{5}{12};\ \ x = \frac{(40)(12)}{5} = 96$; ABC is a multiple 8 of 5,12,13 triple, therefore the dia. AC = 8(13) & r= 52. Area of circle = $52^2\pi$

Area of Hex= $6\left(\left(\frac{104}{\sqrt{3}}\right)^2 \sqrt{3}/_4\right) = 2\sqrt{3}(52)^2$

Bounded Area = $2\sqrt{3}(52)^2 - 52^2\pi$

Ratio: $\left(2\sqrt{3}(52)^2 - 52^2\pi\right)/2\sqrt{3}(52)^2$=

$= \left(2\sqrt{3} - \pi\right) : 2\sqrt{3}$