Solutions:

1. E
   \[ x < -1 \text{ or } x > 1, x \neq 2 \]
   \[
   \frac{(x^2+3x+2)(x-1)}{(x+2)} = \frac{(x+1)(x+2)(x-1)}{x+2} = (x + 1)(x - 1). \]
   Setting this expression greater than zero means that \( x > 1 \) or \( x < -1 \). But \( x \) also must not equal 2.

2. B
   \[ 4\log_2 x + 2 + \log_4 x = 11 \]
   \[ 4\log_2 x + \frac{1}{2}\log_2 x = 9 \]
   \[ 9 \]
   \[ \frac{9}{2} \log_2 x = 9 \]
   \[ \log_2 x = 2 \]
   \[ x = 4 \]

3. A
   The rectangle will have dimension \( x \) by \( y \), where the perimeter of fence is \( 2x+y \). Since this equals 10, \( y=10-2x \). The area of the rectangle is \( A=xy=x(10-2x)=10x-2x^2 \). This is a parabola opening downward, and it has vertex at \( x=5/2 \), implying that \( y=5 \). The area, then, is \( 5\times5/2=25/2=12.5 \), which is closest to 10.

4. B
   80% of 4 miles = 3.2 miles of easy trail; this will take 1.6 hours = 96 minutes
   20% of 4 miles = .8 miles of difficult trail; this will take 32 minutes
   In total, we have 128 minutes, which means that they should leave by 11:52

5. C
   Since the triangle is isosceles, two of the sides must be equal. Setting the sides pairwise equal gives:
   Case 1: \( 2x=x+6 \), so \( x=6 \). This gives sides of 12,12,7, which is acceptable.
   Case 2: \( 2x=x+1 \), so \( x=1 \). This would give a triangle of sides 2,2,7 - not a possible triangle.
   Case 3: \( x+6=x+1 \), which is not possible.

6. D
   \[ \sqrt{3^2 + 4^2 + \sqrt{x^2 + 1}} = 7 \]
   \[ 5 + \sqrt{x^2 + 1} = 7 \]
   \[ \sqrt{x^2 + 1} = 2 \]
   \[ x^2 + 1 = 4 \]
   \[ x^2 = 3 \]
   \[ x = \sqrt{3} \text{ or } -\sqrt{3} \], of which we take the positive value.
7. D

\[(1 - i)^2 = 1 - 2i - 1 = -2i, \text{ so } (1 - i)^6 = (-2i)^3 = 8i\]

8. D

\[x^2 - 4x + 4y^2 + 24y + 36 = 0\]
\[x^2 - 4x + 4 + 4(y^2 + 6y + 9) = 4\]
\[(x - 2)^2 + 4(y + 3)^2 = 4\]
\[\frac{(x-2)^2}{4} + (y + 3)^2 = 1. \text{ This describes an ellipse.}\]

9. A

The length of the major axis is equal to 2a. In the above conic, \(a^2=4\), so \(a=2\); thus, the length of the major axis is 4.

10. B

\[\sum_{n=1}^{127} \log_2 \frac{n}{n+1} = \log_2 \frac{1}{2} + \log_2 \frac{2}{3} + \log_2 \frac{3}{4} + \ldots + \log_2 \frac{127}{128}\]

[\[= \log_2 \left( \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdot \ldots \cdot \frac{127}{128} \right) = \log_2 \frac{1}{128} = -7\]

11. E

DE must equal the difference of BD and BE, so DE = x+4. Thus,
\[(x)(3x) = (x+6)(x+4)\]
\[3x^2 = x^2 + 10x + 24\]
\[2x^2 - 10x - 24 = 0\]
\[x^2 - 5x - 12 \rightarrow x = \frac{5+\sqrt{73}}{2} \text{ (since } x>0)\]

12. C

Area = \(\frac{1}{2} \times \text{ Apothem} \times \text{ Perimeter}\), so
\[a = 2 \times \frac{A}{P} = 2 \times \frac{(\log_2 3)^2 + \log_2 243 + 4}{\log_2 9 + 2} = 2 \times \frac{(\log_2 3 + 1)(\log_2 3 + 4)}{2(\log_2 3 + 1)}\]
\[= \log_2 3 + 4\]

13. E

The x value for which the function is at a minimum is given by \(-\frac{b}{2a} = 2\), so the minimum value is 12-24+4 = -8.
14. D

These inequalities form a trapezoid. The line \( y \geq \frac{x}{2} \) intercepts the upper and lower boundaries of the region at (4,2) and (10,5). The height of the trapezoid is 3. Thus, the area of the trapezoid = \( \frac{1}{2} \times 3 \times (4 + 10) = \frac{1}{2} \times 3 \times 14 = 21 \).

15. D

The difference in their speeds is \((4x+3)-(3x+2) = x+1\) miles per hour. Thus, the time taken for Ankie to make up the half mile is \( \frac{5}{x+1} = \frac{1}{2x+2} \).

16. A

\[ 2l + 2w = 1 + 4lw \]
\[ 2l + l = 1 + 2l^2 \]
\[ 3l = 2l^2 + 1 \]
\[ 0 = 2l^2 - 3l + 1 = (2l - 1)(l - 1) \rightarrow l = 1 \text{ is longest} \]

17. B

\[ .45 = \frac{45}{99} \] since it is an infinite geometric sequence with first term = .45 and ratio \( = \frac{1}{100} \). \( \frac{45}{99} = \frac{5}{11} \)

18. E

Using log rules, we simplify the equation to \[ \ln \left( \frac{5x}{5} \right) - \ln(x + 1) = \ln(x) - \ln(x + 1) = \ln \left( \frac{x}{x+1} \right) = 4 \]. So \( e^4 = \frac{x}{x+1} \) and \( x = \frac{e^4}{1-e^4} \), but this is negative, so there is no solution.

19. B

\[ G(s) = \frac{(2s+1)}{1+\frac{K}{2s+1}} = \frac{(2s+1)^2}{2s+1+K} \]. The denominator is just \( 2s+(1+K) \); its roots are given by 
\[ 2s+(1+K) = 0 \rightarrow s = \frac{1}{2}(-K - 1) \]. We want this to be greater than zero, which will be true when \( 0 < -\frac{K}{2} - \frac{1}{2} \rightarrow K < -\frac{1}{2} \rightarrow K < -1 \)

20. B

The population will double six times in three hours, so you have \( 5 \times (2)^6 = 320 \)

21. D
\[ A = k \cdot B \cdot C \]. We can solve for \( k \) using the values given: \( 5 = k \cdot 4 \cdot 1 \rightarrow k = \frac{5}{4} \).

So \( A = \frac{5}{4} \cdot 2 \cdot 8 = 20 \).

22. D

\( \text{III only: } * \text{ is not commutative, since } A*B=-(B*A). * \text{ is also not associative: for example, } (1*2)*2 = -3*2=5, \text{ while } 1*(2*2)=1*0=1. \) The final statement is equal to \( (0)*2=-4 \)

23. E

Multiplying the two matrices, we get the following equations: \( x^2 + 2 = 3 \) and \( 5x^3 = -5 \)

\[ \Rightarrow X=-1 \]

24. E

The area of a hexagon is \( \frac{3s^2\sqrt{3}}{2} = 18\sqrt{3} \)

25. A

The cylinder with a height of 5 and volume of 30 has a radius of \( r \), found by \( V = \pi r^2 \cdot h \rightarrow 30\pi = \pi \cdot r^2 \cdot 5 \rightarrow r = \sqrt{6}. \) The volume of the sphere with a radius of this \( r \) is \( \frac{4}{3} \cdot \pi \cdot \sqrt{6} = 8\pi \sqrt{6} \)

26. B

\[ \text{volume} = l \cdot w \cdot h = 2 \cdot (12 - 4) \cdot (16 - 4) = 2 \cdot 8 \cdot 12 = 192 \]

27. B

\((A \cup \bar{A})\) is simply everything, so the intersection will just be the second term.

The union of everything that is not in \( B \) but in \( A \) with everything that is in both \( A \) and \( B \) is just \( A \).

28. C

You must make up a half a mile, and the difference in your speeds is 2 mph. It will thus take you a quarter of an hour to catch up, or 15 minutes.

29. C

The \( x \) value of the maximum is given by \( -\frac{b}{2a} = -\frac{6}{-4} = \frac{3}{2}. \) Plugging this in to find \( y \) gives \(-2 \cdot \frac{9}{4} + 9 - 13 = -8.5 \)
30. B

In standard form, the equation of this ellipse is $1 = \frac{b y^2}{9a} + \frac{x^2}{9}$. We want the denominator of the $y^2$ term to be 16, so that our major radius is 4. Thus, $\frac{9a}{b} = 16 \rightarrow \frac{a}{b} = \frac{16}{9}$. 