Solutions:

1. E

$x < -1 \; or \; x > 1, x \neq 2$

$\frac{(x^2+3x+2)(x-1)}{(x+2)} = \frac{(x+1)(x+2)(x-1)}{x+2} = (x+1)(x-1)$. Setting this expression greater than zero means that $x > 1 \; or \; x < -1$. But x also must not equal 2.

2. B

$$4\log_2 x + 2 + \log_4 x = 11$$

$$4\log_2 x + \frac{1}{2}\log_2 x = 9$$

$$\frac{9}{2}\log_2 x = 9$$

$$\log_2 x = 2$$

$$x = 4$$

3. A

The rectangle will have dimension x by y, where the perimeter of fence is 2x+y. Since this equals 10, y=10-2x. The area of the rectangle is A=xy=x(10-2x)=10x-2x^2. This is a parabola opening downward, and it has vertex at x=5/2, implying that y=5. The area, then, is 5*5/2=25/2=12.5, which is closest to 10.

4. B

80% of 4 miles = 3.2 miles of easy trail; this will take 1.6 hours = 96 minutes
20% of 4 miles = .8 miles of difficult trail; this will take 32 minutes
In total, we have 128 minutes, which means that they should leave by 11:52

5. C

Since the triangle is isosceles, two of the sides must be equal. Setting the sides pairwise equal gives:
Case 1: 2x=x+6, so x=6. This gives sides of 12,12,7, which is acceptable.
Case 2: 2x=x+1, so x=1. This would give a triangle of sides 2,2,7 - not a possible triangle.
Case 3: x+6=x+1, which is not possible.

6. D

$$\sqrt{3^2+4^2} + \sqrt{x^2+1} = 7$$

$$5 + \sqrt{x^2+1} = 7$$

$$\sqrt{x^2+1} = 2$$

$$x^2 + 1 = 4$$

$$x^2 = 3$$

$x = \sqrt{3} \; or -\sqrt{3}$, of which we take the positive value.

7. D

$$(1-i)^2 = 1-2i-1 = -2i, so\ (1-i)^6 = (-2i)^3 = 8i$$

8. D

$$x^2 - 4x + 4y^2 + 24y + 36 = 0$$
$$x^2 - 4x + 4 + 4(y^2 + 6y + 9) = 4$$
$$(x-2)^2 + 4(y+3)^2 = 4$$

$\frac{(x-2)^2}{4} + (y+3)^2 = 1$. This describes an ellipse.

9. A

The length of the major axis is equal to 2a. In the above conic, a^2=4, so a=2; thus, the length of the major axis is 4.

10. B

$$\sum_{1}^{127} \log_2 \frac{n}{n+1} = \log_2 \frac{1}{2} + \log_2 \frac{2}{3} + \log_2 \frac{3}{4} \ldots + \log_2 \frac{127}{128}$$
$$= \log_2 \frac{1}{2} * \frac{2}{3} * \frac{3}{4} * \ldots * \frac{127}{128} = \log_2 \frac{1}{128} = -7$$

11. E

DE must equal the difference of BD and BE, so DE = x+4. Thus,

$$(x)(3x) = (x+6)(x+4)$$
$$3x^2 = x^2 + 10x + 24$$
$$2x^2 - 10x - 24 = 0$$
$$x^2 - 5x - 12 \to x = \frac{5+\sqrt{73}}{2} \text{ (since x>0)}$$

12. C

Area = ½ * Apothem * Perimeter, so

$$a = 2 * \frac{A}{P} = 2 * \frac{(\log_2 3)^2 + \log_2 243 + 4}{\log_2 9 + 2} = 2 * \frac{(\log_2 3 + 1)(\log_2 3 + 4)}{2(\log_2 3 + 1)}$$
$$= \log_2 3 + 4$$

13. E

The x value for which the function is at a minimum is given by $-\frac{b}{2a} = 2$, so the minimum value is 12-24+4 = -8.

14. D

These inequalities form a trapezoid. The line $y \geq \frac{x}{2}$ intercepts the upper and lower boundaries of the region at (4,2) and (10,5). The height of the trapezoid is 3. Thus, the area of the trapezoid $= \frac{1}{2} * 3 * (4 + 10) = \frac{1}{2} * 3 * 14 = 21.$

15. D

The difference in their speeds is (4x+3)-(3x+2) = x+1 miles per hour. Thus, the time taken for Ankie to make up the half mile is $\frac{.5}{x+1} = \frac{1}{2x+2}$

16. A

$$2l + 2w = 1 + 4lw$$
$$2l + l = 1 + 2l^2$$
$$3l = 2l^2 + 1$$
$$0 = 2l^2 - 3l + 1 = (2l - 1)(l - 1) \rightarrow l = 1 \text{ is longest}$$

17. B

$\overline{.45} = \frac{45}{99}$ since it is an infinite geometric sequence with first term = .45 and ratio = 1/100. $\frac{45}{99} = \frac{5}{11}$

18. E

Using log rules, we simplify the equation to $\ln\left(\frac{5x}{5}\right) - \ln(x + 1) = \ln(x) - \ln(x + 1) = \ln\left(\frac{x}{x+1}\right) = 4$. So $e^4 = \frac{x}{x+1}$, and $x = \frac{e^4}{1-e^4}$, but this is negative, so there is no solution.

19. B

$G(s) = \frac{(2s+1)}{1+\frac{K}{2s+1}} = \frac{(2s+1)^2}{2s+1+K}$. The denominator is just 2s+(1+K); its roots are given by 2s+(1+K) = 0 → $s = \frac{1}{2}(-K - 1)$. We want this to be greater than zero, which will be true when $0 < -\frac{K}{2} - \frac{1}{2} \rightarrow \frac{K}{2} < -\frac{1}{2} \rightarrow K < -1$

20. B

The population will double six times in three hours, so you have $5 * (2)^6 = 320$

21. D

$A = k * B * C$. We can solve for k using the values given: $5 = k * 4 * 1 \to k = \frac{5}{4}$.
So $A = \frac{5}{4} * 2 * 8 = 20$.

22. D

III only: * is not commutative, since A*B= -(B*A). * is also not associative: for example, (1*2)*2 = -3*2=5, while 1*(2*2)=1*0=1. The final statement is equal to (0)*2=-4

23. E

Multiplying the two matrices, we get the following equations: $x^2 + 2 = 3$
and $5x^3 = -5$

- ➔ X=-1

24. E

The area of a hexagon is $\frac{3s^2\sqrt{3}}{2} = 18\sqrt{3}$

25. A

The cylinder with a height of 5 and volume of 30 has a radius of r, found by $V = \pi r^2 * h \to 30\pi = \pi * r^2 * 5 \to r = \sqrt{6}$. The volume of the sphere with a radius of this r is $\frac{4}{3} * \pi * \sqrt{6}^3 = 8\pi\sqrt{6}$

26. B

$$volume = l * w * h = 2 * (12 - 4) * (16 - 4) = 2 * 8 * 12 = 192$$

27. B

$(A \cup \bar{A})$is simply everything, so the intersection will just be the second term. The union
of everything that is not in B but in A with everything that is in both A and B is just A.

28. C

You must make up a half a mile, and the difference in your speeds is 2 mph. It will thus take you a quarter of an hour to catch up, or 15 minutes.

29. C

The x value of the maximum is given by $-\frac{b}{2a} = -\frac{6}{-4} = \frac{3}{2}$. Plugging this in to find y gives $-2 * \frac{9}{4} + 9 - 13 = -8.5$

30. B

In standard form, the equation of this ellipse is $1 = \frac{by^2}{9a} + \frac{x^2}{9}$. We want the denominator of the y^2 term to be 16, so that our major radius is 4. Thus, $\frac{9a}{b} = 16 \rightarrow \frac{a}{b} = \frac{16}{9}$.