

Solutions:

$$1. \frac{1}{1+i} \cdot \frac{1-i}{1-i} = \frac{1-i}{2} \quad D$$

$$2. \text{When you solve for } y \text{ you see this is exponential so } y\text{-axis!! } y = 10^x \quad B$$

3. Use sum of roots and product of roots. Imaginary roots come in conjugate pairs.

$$2 \pm i\sqrt{3} \rightarrow \frac{-b}{a} = 4 \rightarrow \frac{c}{a} = 7 \rightarrow 7 - 4 = 3 \quad C$$

$$4. \begin{aligned} x + y = 1 &\rightarrow \frac{x}{y} = \frac{1}{\sqrt{2}} \rightarrow y = x\sqrt{2} \rightarrow x + x\sqrt{2} = 1 \rightarrow x = \frac{1}{1+\sqrt{2}} = \frac{1-\sqrt{2}}{-1} = \sqrt{2}-1 \quad D \\ \sqrt{2}(\sqrt{2}-1) &= 2 - \sqrt{2} \end{aligned}$$

$$5 = A(3x^2 - 7) - (x-2)(Bx+C) \rightarrow 5A = 5 \rightarrow A = 1$$

$$5. 5 = 3x^2 - 7 - Bx^2 - xC + 2Bx + 2C \rightarrow B = 3 \rightarrow 2C = 12 \rightarrow C = 6 \quad B \\ 1 + 3 - 6 = -2$$

6. We are looking for the y-coordinate of the vertex. The x-coordinate is $-b/2a$. Plug it in and solve for y!!

$$y!! \quad a\left(\frac{-b}{2a}\right)^2 + b\left(\frac{-b}{2a}\right) + c = \frac{b^2 - 2b^2 + 4ac}{4a} = \frac{-b^2 + 4ac}{4a} \rightarrow D$$

$$7. \frac{1}{1-\frac{2}{9}} = \frac{9}{7} \quad D$$

$$x = \frac{y+3}{-2} \rightarrow 2\left(\frac{y+3}{-2}\right)^2 + 6\left(\frac{y+3}{-2}\right) + 5y + 1 = 0$$

$$8. \frac{y^2 + 6y + 9}{2} - 3y - 9 + 5y + 1 = 0 \quad C$$

$$y^2 + 6y + 9 - 6y - 18 + 10y + 2 = 0 \rightarrow y^2 + 10y - 7 = 0$$

$$9. (a^2 - 25b^2)^4 \text{ so } 4+1=5 \text{ terms } B$$

$$10. \quad 4 - n = \sqrt{n-2} \rightarrow 16 - 8n + n^2 = n - 2 \rightarrow n^2 - 9n + 18 = 0$$

$$(n-3)(n-6) = 0 \rightarrow n = 3 \quad \text{6 is extraneous!!} \quad \text{A}$$

$$11. \quad \frac{3-\sqrt{2}}{4+\sqrt{2}} \cdot \frac{4-\sqrt{2}}{4-\sqrt{2}} = \frac{12-7\sqrt{2}+2}{14} = \frac{14-7\sqrt{2}}{14} = \frac{2-\sqrt{2}}{2} \quad \text{A}$$

12. Draw a picture. Draw a radius from the center to the tangent point. This creates similar triangles. This creates a $\frac{1}{2}$ ratio so radius of smaller circle is 6. The radius of larger circle is 18 B

$$13. \quad \frac{xy(x+y^{-1})}{xy(x^{-1}+y)} = 13 \rightarrow \frac{x^2y+x}{y+xy^2} = 13 \rightarrow \frac{x(xy+1)}{y(1+xy)} = 13 \rightarrow x = 13y$$

$$14y \leq 100 \rightarrow 0 < y \leq 7$$

Therefore 7 solutions: C

$$14. \quad \frac{1}{i^{2017}} = \frac{i^3}{i^{2020}} = -i \quad \text{B}$$

$$15. \quad \frac{4^{-2}a^3b^{-3}}{2^3(ab^{-2})^2} \cdot \frac{(a^3b^3)^{-1}}{(8a)^{-3}} = \frac{ab}{2^7} \cdot \frac{2^9}{b^3} = \frac{4a}{b^2} \quad \text{D}$$

16. Draw a picture!! ZW=MU=MZ. We now see that triangle MWZ is equilateral. This makes the angle we are looking for equal to $90-60=30$ degrees. Answer C

17. Since the coefficient of y equal 1, the factors of k must be one apart. So (1,2), (2,3)...(9,10). We stop here because (10, 11) violates the limits on k. So 9 possibilities. Answer C

$$18. \quad {}_{26}C_2 - {}_{13}C_2 - 13 = 325 - 78 - 13 = 234 \quad \text{C}$$

19. The slope of the segment is $\frac{264}{45}$. If prime factor these you have only one common factor, which is 3. This leaves $\frac{88}{15}$. So you had 15 to each x coordinate and 88 to each y coordinate. This gives 2 additional answers besides the endpoints (20,107) and (35,195). Answer B

20. Draw picture!! Extend LU and MZ to meet at X. Angle X will = 30 degrees so XL = 6 and XU = 10 so

$$\text{MU} = \frac{10}{\sqrt{3}} \quad \text{D}$$

$$i = x^2 + 2xyi - y^2 \rightarrow x^2 - y^2 = 0 \rightarrow 2xy = 1 \rightarrow y = \frac{1}{2x}$$

21.

$$x^2 - \frac{1}{4x^2} = 0 \rightarrow 4x^4 - 1 = 0 \rightarrow x^2 = \pm \frac{1}{2} \rightarrow y^2 = \pm \frac{1}{2}$$

They do have the same sign so $1/2 + 1/2 = 1$ B

22. For a matrix to not have an inverse its determinant must equal zero. Answer D

23. x- intercept is $5/3$ and the y-intercept is 5. Add them together and you get $20/3$ D

24. To make the vertex lie on the x-axis you need one double root so set the discriminant equal to zero and solve. $b^2 - 4ac = 0 \rightarrow 4n^2 - 4(n^2 - 9n + 9) = 0 \rightarrow 36n - 36 = 0 \rightarrow n = 1$ B

25. Factor as difference of squares

$$\left[\left(\frac{e^y + e^{-y}}{2} \right) - \left(\frac{e^y - e^{-y}}{2} \right) \right] \left[\left(\frac{e^y + e^{-y}}{2} \right) + \left(\frac{e^y - e^{-y}}{2} \right) \right] = e^{-y} \cdot e^y = 1 \quad C$$

26. Reorder the terms and then factor as difference of squares twice!!

$$a^2 - 2ac + c^2 - b^2 + 2bx - x^2 = (a - c)^2 - (b - x)^2 = (a - c + b - x)(a - c - b + x) \quad A$$

$$27. e^3 = 6(12e)^2 \rightarrow e = 864 \quad D \quad e^3 = 6(12e)^2 \rightarrow e = 864 \quad D$$

28. Diagonals intersect at right angles. Use Pythagorean theorem with half the diagonal length as sides:

$$4^2 + \frac{x^2}{4} = 169 \rightarrow \frac{x^2}{4} = 153 \rightarrow x^2 = 612 \rightarrow 24 < x < 25 \quad D$$

$$29. \quad x^2 - 8x + 16 + y^2 = 65 \rightarrow (x - 4)^2 + y^2 = 81$$

$$x^2 - 16x + 64 + y^2 - 10y + 25 = -73 + 89 \rightarrow (x - 8)^2 + (y - 5)^2 = 16$$

The first is a circle with center (4,0) and radius 9. The second is a circle center at (8,5) and radius 4. If you graph them you see that the answer is 2. You could also solve the first equation for y-squared and substitute into the second. This will lead to a quadratic which also tells you there are 2 solutions. C

30. If the sum of the absolute values equals zero then each absolute value is zero. Solve the system and

$$2m + n = -6$$

plug in!! $2m - n = 14$

C

$$4m = 8 \rightarrow m = 2 \rightarrow n = -10 \rightarrow |2 - 10| = 8$$

Answers:

1. D
2. B
3. C
4. D
5. B
6. D
7. D
8. C
9. B
10. A
11. A
12. B
13. C
14. B
15. D
16. C
17. C
18. C
19. B
20. D
21. B
22. D
23. D
24. B
25. C
26. A
27. D
28. D
29. C
30. C