

Where applicable, "E) NOTA" indicates that none of the above answers is correct.

1. Solve for x : $2\log_3 x = 2\log_3(1-a) + 2\log_3(1+a) - \log_3\left(\frac{1}{a} - a\right)^2$

- A) a B) $\frac{1}{a}$ C) $2a$ D) $a+1$ E) NOTA

2. Which of the following numbers does not have a square root of the form $x + y\sqrt{2}$, where x and y are both positive integers?

- A) $17 + 12\sqrt{2}$ B) $22 + 12\sqrt{2}$ C) $38 + 12\sqrt{2}$ D) $54 + 12\sqrt{2}$ E) NOTA

3. Simplify: $\frac{\log_3 \sqrt{243\sqrt{81^3 3}}}{\log_2 \sqrt[4]{64} + \ln e^{-13}}$

- A) $-\frac{29}{68}$ B) $-\frac{43}{138}$ C) $-\frac{15}{31}$ D) $-\frac{15}{138}$ E) NOTA

4. Solve for $x > 0$: $5^{2+4+6+\dots+2x} = 0.04^{-28}$

- A) 6 B) 12 C) 14 D) 7 E) NOTA

5. Evaluate: $\frac{1}{\log_2 36} + \frac{1}{\log_3 36}$

- A) $\frac{1}{2}$ B) -2 C) $\ln 6$ D) $e^{\frac{1}{36}}$ E) NOTA

6. If f is a function of x only while g is a function of y only, determine f such that $\log f + \log g = \log(1+x+xy+y)$.

- A) $1+x$ B) x C) $2+x$ D) $2+2x$ E) NOTA

7. If $(ax)^{\log a} = (bx)^{\log b}$, where $a, b > 0$; $a, b \neq 1$; and $a \neq b$; find x in terms of a and b .

- A) $a+b$ B) $\frac{a}{b}$ C) $\frac{1}{ab}$ D) $a-b$ E) NOTA

8. Solve: $2 \cdot 5^{x+1} = 1 + \frac{3}{5^x}$

- A) $-1 + \log_5 3$ B) $-1 + \log_3 5$ C) $1 + \log_5 3$ D) $1 + \log_3 5$ E) NOTA

9. Given that $(\log_3 p)^2 = \log_3(p^2)$ and $\log_3(p+q) = \log_3 p + \log_3 q$, evaluate $\frac{q}{p}$.

- A) 8 B) $\frac{3}{2}$ C) $\frac{1}{8}$ D) $\frac{2}{3}$ E) NOTA

10. Which of the following is divisible by 9?

- A) $10^{2316} + 6$ B) $10^{2316} + 7$ C) $10^{2316} + 8$ D) $10^{2316} + 9$ E) NOTA

11. Simplify: $(5^n)(12^n)(15^n)$

- A) 15^{2n} B) 20^{2n} C) 24^{2n} D) 30^{2n} E) NOTA

12. If $x^2yz^3 = 7^4$ and $xy^2 = 7^5$, evaluate xyz .

- A) 7 B) 7^2 C) 7^3 D) 7^4 E) NOTA

13. Solve: $2^{\log_4 8} = 16^x$

- A) $\frac{3}{8}$ B) $\frac{1}{2}$ C) $2^{\frac{3}{52}}$ D) $2^{\frac{1}{8}}$ E) NOTA

14. Simplify: $\left(\frac{x^{4a-3b-c}}{x^{3a-b}}\right)x^{3c+2b-a}$

- A) x^{4c} B) x^{-4c} C) x^{2c} D) x^{-2c} E) NOTA

15. Consider the equation $\sqrt{\sqrt{x^2+2}+x} - \sqrt{\sqrt{x^2+2}-x} = 2 \cdot \sqrt[4]{2}$. How many rational numbers are solutions to this equation?

- A) 0 B) 1 C) 2 D) 3 E) NOTA

16. If $\frac{4^{3x}}{2^{x+y}} = 128$ and $\frac{5^{6x-y}}{25^{y-14x}} = 25$, find the numerical value of xy .

- A) 12 B) 8 C) -12 D) -8 E) NOTA

17. If $f(x) = 3 + \ln(x-2)$, $g(x) = 3 + \ln x$, and $h(x) = \ln x$, which of the following are true?

- I) The graph of $y = f(x)$ is the graph of $y = g(x)$ shifted two units to the right.
 II) The graph of $y = f(x)$ is the graph of $y = g(x)$ shifted two units to the left.
 III) The graph of $y = f(x)$ is the graph of $y = h(x)$ shifted two units to the right and three units upward.
 IV) The graph of $y = f(x)$ is the graph of $y = h(x)$ shifted two units to the left and three units downward.

- A) I only B) I and IV only C) III only D) I and III only E) NOTA

18. Determine the positive integer n such that $\frac{1}{3^{13}} + \frac{1}{3^{10}} + \frac{1}{3^8} = \frac{n}{3^{14}}$.

- A) 3 B) 9 C) 271 D) 813 E) NOTA

19. If $f\left(\frac{1}{5}\log_5 x\right) = x^2$, then what is $f(x)$?

- A) 5^{25x} B) $\frac{1}{10}(\log_5 x)^2$ C) $\frac{1}{5}\log_5\left(\frac{1}{5}\log_5 x\right)$ D) 5^{10x} E) NOTA

20. If $3^{6x} = 30$, which of the following are true?

- I) $x = \frac{\ln 30}{6 \ln 3}$ II) $x = \frac{1}{6}\left(1 + \frac{\ln 10}{\ln 3}\right)$ III) $x = \log_{729} 30$

- A) I only B) III only C) I and II only D) I and III only E) NOTA

21. Evaluate: $\prod_{n=2}^{63} \log_n(n+1)$

- A) 6 B) 5.75 C) 5.5 D) 5 E) NOTA

22. What is the sum of the last two digits (the tens' and ones' digits) in the expansion of 7^{2016} ?

- A) 1 B) 7 C) 13 D) 14 E) NOTA

23. Suppose m and n are positive integers such that $\sqrt{m} - \sqrt{n} = 2\sqrt{2 - \sqrt{3}}$. Evaluate $\frac{m}{n}$.

- A) $\frac{9}{8}$ B) 3 C) $\frac{9}{4}$ D) 6 E) NOTA

24. Which function would be a model representing exponential decay?

- A) $f(x) = (0.5x)^2$ B) $f(x) = 0.5(2)^x$ C) $f(x) = 1.5\left(\frac{10}{11}\right)^x$ D) $f(x) = 0.9(1.01)^x$ E) NOTA

25. Suppose Jackson and Brooks are painting a house together. If Jackson can paint the house by himself in $\log_2 e$ hours, and if Brooks can paint the house by himself in $\log_5 e^2$, how many hours will it take the brothers to paint the house, working together and independently?

- A) $\frac{\log_2 e + \log_5 e^2}{2}$ B) $\ln 10$ C) $\log_{10} e^3$ D) $\log_{20} e^2$ E) NOTA

26. A circle has a radius of length $\ln\sqrt{a^3}$ and a circumference of $\ln(b^{2\pi})$. Evaluate $\log_a b$.

- A) $\frac{3}{2}$ B) $\frac{2}{3}$ C) π D) $\frac{1}{2\pi}$ E) NOTA

27. For the function $f(x) = 0.25(0.75)^x$, a one unit increase in x yields what percent increase or decrease in $f(x)$?

- A) 75% increase B) 25% increase C) 75% decrease D) 25% decrease E) NOTA

28. Honest Abe Lincoln takes time off from his homework to split some logs for the fire. He produces $\frac{1}{2}\log 20$ pounds of wood on Monday, $\frac{1}{3}\log 30$ pounds of wood on Tuesday, $\frac{1}{4}\log 40$ pounds of wood on Wednesday, $\frac{1}{5}\log 50$ pounds of wood on Thursday, and $\frac{1}{6}\log 60$ pounds of wood on Friday. On which day did Honest Abe Lincoln produce the second largest amount of wood?

- A) Monday B) Tuesday C) Wednesday D) Thursday E) Friday

29. A 200 gram sample of lawrencium is left in a container from 8 AM one morning until 2 pm the next day. If the remaining mass of the sample at that time was 25 gram, what is the half-life of lawrencium?

- A) 5 hours B) 10 hours C) 24 hours D) 30 hours E) NOTA

30. Which of the following is true concerning the graphs of $y = \log_2 x + 3$ and $y = 2^x$?

- A) the graphs do not intersect B) the graphs intersect exactly once
C) the graphs intersect exactly twice D) the graphs intersect exactly thrice
E) NOTA