Where applicable, “E) NOTA” indicates that none of the above answers is correct.

1. Given \(\log_a b = c\) where \(a, b\) are positive integers and \(a > 1\). Which of the following equations is not true for all such \(a, b,\) and \(c\)?

A) \(a^c = b\)  
B) \(\ln \frac{b}{a} = c\)  
C) \(\ln b = \frac{c}{\ln a}\)  
D) \(a^{\log_a b} = a^c\)  
E) NOTA

2. Which of the following is an equation of the graphed function?

A) \(y = -\log_2 x\)  
B) \(y = -2^x\)  
C) \(y = 2^{-x}\)  
D) \(y = \log_{-2} x\)

3. Suppose that \(\ln x\) and \(\log x\) are both integers. How many possible values of \(x\) are there?

A) 0  
B) 1  
C) 2  
D) 3  
E) NOTA

4. Find the sum of the solutions of the equation \(\log x + \log(x + 30) = 3\).

A) 20  
B) −30  
C) 30  
D) 13.5  
E) NOTA

5. Find the median of \(\ln e, \log 0.01, \log_4 2, e^0\)

A) \(\frac{1}{8}\)  
B) 1  
C) \(\frac{3}{2}\)  
D) \(\frac{3}{4}\)  
E) NOTA

6. Find the domain of \(y = \log(4 + 3x - x^2)\)

A) \(-1 < x < 4\)  
B) \(x < -1\) or \(x > 4\)  
C) \(x\) is all real numbers  
D) \(x > -\frac{3}{8}\)  
E) NOTA

7. A sum of money is to be invested in an account whose interest is compounded continuously for 10 years. What would the annual rate of interest be if the final amount is twice the original?

A) \(\frac{\ln 10}{2}\)  
B) \(\ln 20\)  
C) \(\ln 2 - \ln 10\)  
D) \(\ln 5\)  
E) NOTA

8. Evaluate \(\left(x - \frac{2}{x}\right)^2\) when \(x = \frac{\sqrt{5}}{5}\)

A) \(\frac{576}{5}\)  
B) \(\frac{242}{5}\)  
C) \(\frac{1152}{25}\)  
D) \(\frac{496}{25}\)  
E) NOTA
9. If \( a^2 = a + 2 \) find \( a^3 \) in terms of the first power of \( a \).
   
   A) \( a + 4 \)  
   B) \( 2a + 8 \)  
   C) \( 3a + 2 \)  
   D) \( 27a + 8 \)  
   E) NOTA

10. If \( 5^{2015} - 5^{2014} + 5^{2013} = k \cdot 5^{2012} \), what is the value of \( k \)?

   A) 4  
   B) 21  
   C) 105  
   D) 55  
   E) NOTA

11. If \( L > 1 \), then \( \sqrt[3]{L \sqrt{L^{3/2}}} = ? \)

   A) \( L^{\frac{1}{27}} \)  
   B) \( L^{\frac{1}{5}} \)  
   C) \( L^{\frac{1}{3}} \)  
   D) \( L^{\frac{13}{27}} \)  
   E) NOTA

12. There exists positive integers \( m, n, \) and \( p \) with greatest common factor 1, such that

   \[ m \log_{200} 5 + n \log_{200} 2 = p \]

   What is \( m + n + p \)?

   A) 6  
   B) 7  
   C) 8  
   D) 9  
   E) NOTA

13. Find the positive number \( n \) such that \( n^{\log_{17} 89} = 89^2 \).

   A) \( \sqrt{17} \)  
   B) 17  
   C) 289  
   D) \( \frac{89}{34} \)  
   E) NOTA

14. \( N = \sqrt{\frac{1}{10^\log_{1000} 1}} \). Find \( \log N \).

   A) \( 10^{\frac{2}{3}} \)  
   B) \( \frac{2}{3} \)  
   C) 10  
   D) \( \frac{3}{2} \)  
   E) NOTA

15. There exists two positive integers \( a, b \) such that \( 2\sqrt{2} - \sqrt{3} = \sqrt{a} - \sqrt{b} \). Find \( ab \).

   A) 24  
   B) 12  
   C) 8  
   D) 48  
   E) NOTA

16. In the expression \( ca^b - d \), the values of \( a, b, c \) and \( d \) are 0, 1, 2 and 3, although not necessarily in that order. What is the maximum possible value of the result?

   A) 5  
   B) 6  
   C) 8  
   D) 9  
   E) NOTA

17. What is the sum of the solutions of \( x^{x\sqrt{x}} = (x\sqrt{x})^x \)?

   A) 0  
   B) \( \frac{3}{2} \)  
   C) \( \frac{9}{4} \)  
   D) \( \frac{13}{4} \)  
   E) NOTA
18. Suppose that \(13!\) is written as \(2^p3^q5^r7^s11^t13^u\), where all exponents are positive integers. What is the value of \(p - q + r - s + t - u\)?

A) 20  B) 6  C) -6  D) 5  E) NOTA

19. The graphs of \(y = \log_3 x\) and \(y = \log_3 x^2 + 1\) intersect in exactly one point \((a, b)\). What is \(a + b\)?

A) \(-\frac{2}{3}\)  B) \(\frac{1}{3}\)  C) 1  D) 6  E) NOTA

20. Evaluate \(\left(\frac{1}{\log_2 3}\right)^2\left(\frac{1}{\log_3 2}\right)^2\left(\frac{1}{\log_4 3}\right)^2\left(\frac{1}{\log_2 9}\right)^2\).

A) 1  B) \(\frac{1}{2}\)  C) \(\frac{1}{3}\)  D) \(\frac{1}{16}\)  E) NOTA

21. The cardinality of \(\{x|e^{x\ln 5} = 25\}\) is

A) 0  B) 1  C) 2  D) 3  E) NOTA

22. What is the units digit of \(7^{2015}\)?

A) 1  B) 3  C) 7  D) 9  E) NOTA

23. Find the value of \(-\sqrt{5 - \sqrt{5 - \sqrt{5 - \ldots}}}\).

A) \(\frac{1 + \sqrt{21}}{2}\)  B) \(\frac{-1 + \sqrt{21}}{2}\)  C) \(\frac{1 - \sqrt{21}}{2}\)  D) \(\frac{-1 - \sqrt{21}}{2}\)  E) NOTA

24. Determine the coefficient of \(x^3\) in the expansion of \(\left(\frac{x^2 + 2}{4}\right)^{12}\).

A) \(\frac{1}{8}\)  B) 35  C) 99  D) 792  E) NOTA
25. Suppose that $a$ and $b$ are positive real numbers such that
\[ \log_{27} a + \log_9 b = \frac{7}{2} \quad \text{and} \quad \log_{27} b + \log_9 a = \frac{2}{3}. \]
Determine the value of $ab$.

A) 125 \hspace{1cm} B) 243 \hspace{1cm} C) \frac{25}{6} \hspace{1cm} D) 6 \hspace{1cm} E) NOTA

26. In a right triangle the leg lengths are $2^5$ and $5(4)^3$. The length of the hypotenuse can be written in the form $a(2)^b$, where $a$ is not divisible by 2. Find $b + a$.

A) 352 \hspace{1cm} B) 36 \hspace{1cm} C) 5 + \sqrt{26} \hspace{1cm} D) 5 + \sqrt{101} \hspace{1cm} E) NOTA

27. Richter scale magnitude of an earthquake can be calculated using $R = \frac{2}{3} \log \frac{E}{E_0}$, where $E$ is the energy produced and $E_0$ is a constant. How many times more energy does an 8.5 earthquake produce as a 7.1 earthquake?

A) between 0 and 10 \hspace{1cm} B) between 10 and 100 \hspace{1cm} C) between 100 and 1000 \hspace{1cm} D) between 1000 and 10000 \hspace{1cm} E) NOTA

28. If $x$ is such a number that $3x + \frac{1}{2x} = 4$, find the numerical value of $27x^3 + \frac{1}{8x^3}$.

A) 46 \hspace{1cm} B) 48 \hspace{1cm} C) 58 \hspace{1cm} D) 64 \hspace{1cm} E) NOTA

29. Let $x$ and $y$ be positive real numbers such that $\log_x y = -\frac{1}{4}$. Find the value of $\log_x (xy^5) - \log_y \left(\frac{x^2}{\sqrt{y}}\right)$.

A) $\frac{33}{4}$ \hspace{1cm} B) $\frac{29}{4}$ \hspace{1cm} C) $-\frac{31}{4}$ \hspace{1cm} D) $-\frac{35}{4}$ \hspace{1cm} E) NOTA

30. For $f(x) = 6 \log_9 (x - 1) - 4$, calculate $f^{-1}(-2)$.

A) 9 \hspace{1cm} B) 3 \hspace{1cm} C) 1 \hspace{1cm} D) -2 \hspace{1cm} E) NOTA