Where applicable, "E) NOTA" indicates that none of the above answers is correct.

1. Answer B: $\ln\frac{b}{a} = c$. $\ln\frac{b}{a} = \ln b - \ln a$ which is not equivalent to $\log_a b$.

2. Answer B: $y = -2^x$. The graph of $y = -2^x$ has y-intercept at $(0, -1)$ and has points at $(1, -2)$ and $\left(-1, -\frac{1}{2}\right)$. It is a reflection of $y = 2^x$ with respect to the x axis.

3. Answer B: $x = 1$. For $\ln x$ to be an integer x must be $1, e, e^2, \ldots$ For $\log x$ to be an integer x must be $1, 10, 10^2, \ldots$ Therefore x must be 1 for both of those conditions to be true.

4. Answer A: 20.

$$\log x + \log(x + 30) = 3$$
$$\log[x(x + 30)] = 3$$
$$10^3 = x(x + 30)$$
$$1000 = x^2 + 30x$$
$$x^2 + 30x - 1000 = 0$$
$$(x + 50)(x - 20) = 0$$

$x = -50, 20$ Since -50 is extraneous, the sum is 20.

5. Answer D: $\frac{3}{4}$.

$\ln e$, $\log 0.01$, $\log_4 2$, $e^0 = 1, -2, \frac{1}{2}, 1$

Ordered from lowest to highest: $-2, \frac{1}{2}, 1, 1$

Median is $\frac{\frac{1}{2}+1}{2} = \frac{3}{4}$

6. Answer A: $-1 < x < 4$.

$$4 + 3x - x^2 > 0$$
$$x^2 - 3x - 4 < 0$$
$$(x - 4)(x + 1) < 0$$
$$-1 < x < 4$$

7. Answer E: NOTA

$$A = Pe^{rt}$$
$$2 = 1e^{r(10)}$$
$$\ln 2 = 10r$$
$$r = \frac{\ln 2}{10}$$

8. Answer C: $\frac{1152}{25}$.

$$\left(\frac{\sqrt{2}}{5} - \frac{2}{\frac{\sqrt{2}}{5}}\right)^2 = \left(\frac{\sqrt{2}}{5} - \frac{10}{\sqrt{2}}\right)^2 = \left(\frac{2-50}{5\sqrt{2}}\right)^2 = \left(\frac{-48}{5\sqrt{2}}\right)^2 = \frac{48^2}{50} = \frac{1152}{25}$$

9. Answer C: $3a + 2$

$$a^3 = a^2 a = a(a + 2) = a^2 + 2a = (a + 2) + 2a = 3a + 2$$

10. Answer C: 105

$$5^{2015} - 5^{2014} + 5^{2013} = k \cdot 5^{2012}$$

$5^{2012}(5^3 - 5^2 + 5) = 5^{2012}(105)$. Therefore k must be105.

11. Answer D: $L^{\frac{13}{27}}$. $\sqrt[3]{L\sqrt[3]{L\sqrt[3]{L}}} = \left(L\left(L(L)^{\frac{1}{3}}\right)^{\frac{1}{3}}\right)^{\frac{1}{3}} = \left(L\left(L^{\frac{4}{3}}\right)^{\frac{1}{3}}\right)^{\frac{1}{3}} = \left(L(L)^{\frac{4}{9}}\right)^{\frac{1}{3}} = \left(L^{\frac{13}{9}}\right)^{\frac{1}{3}} = L^{\frac{13}{27}}$

12. Answer A: 6

$$m\log_{200} 5 + n\log_{200} 2 = p$$
$$\log_{200} 5^m + \log_{200} 2^n = p$$
$$\log_{200}(5^m 2^n) = \log_{200} 200^p$$
$$5^m 2^n = 200^p$$
$$5^m 2^n = (5^2 2^3)^p$$
$$5^m = 5^{2p} \text{ and } 2^n = 2^{3p}$$
$$m = 2p \quad \text{ and } n = 3p$$

For m, n, p to all be positive integers with GCF of 1, p must be 1.
Therefore $m = 2 \; and \; n = 3$. So $m + n + p = 6$.

13. Answer C.

$$n^{\log_{17} 89} = 89^2$$
$$\log_n n^{\log_{17} 89} = \log_n 89^2$$
$$\log_{17} 89 = 2\log_n 89$$
$$\frac{\ln 89}{\ln 17} = \frac{2\ln 89}{\ln n}$$
$$\frac{1}{\ln 17} = \frac{2}{\ln n}$$
$$\ln n = 2\ln 17$$
$$\ln n = \ln 17^2$$
$$n = 17^2 = 289$$

14. Answer D. $N = \sqrt{\frac{1}{10^{-\log 1000}}} = \sqrt{10^{\log 1000}} = \sqrt{10^3} = 10^{\frac{3}{2}}$. $\log N = \log 10^{\frac{3}{2}} = \frac{3}{2}$.

15. Answer B.

$$2\sqrt{2-\sqrt{3}} = \sqrt{a} - \sqrt{b}$$
$$4\left(2-\sqrt{3}\right) = a - 2\sqrt{ab} + b$$
$$8 - 4\sqrt{3} = a + b - 2\sqrt{ab}$$

If *a* and *b* are to be positive integers then $4\sqrt{3} = 2\sqrt{ab}$ and $16(3) = 4(ab)$.
Therefore $ab = 12$.

16. Answer D. To be a maximum, *d* must be 0 and *a* must not be 1. Testing both 2 and 3 as the exponent *b* yields two sums for $ca^b - d$:
$(1)(2)^3 - 0 = 8$ and $(1)(3)^2 = 9$. The maximum must be 9.

17. Answer D. $x^{x\sqrt{x}} = \left(x\sqrt{x}\right)^x$

$x^{x^{\frac{3}{2}}} = \left(x^{\frac{3}{2}}\right)^x$

$x^{x^{\frac{3}{2}}} = x^{\frac{3x}{2}}$

$x^{\frac{3}{2}} = \frac{3x}{2}$

$x^{\frac{3}{2}} - \frac{3x}{2} = 0$

$x\left(x^{\frac{1}{2}} - \frac{3}{2}\right) = 0$

$x = 0$ and $x^{\frac{1}{2}} = \frac{3}{2}$. $x = 0$ doesn't work, but x=1 (trivial solution) and $x = \frac{9}{4}$ work., so the sum must be $\frac{13}{4}$.

18. Answer B: $13! = 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \cdot 10 \cdot 11 \cdot 12 \cdot 13 =$

$2 \cdot 3 \cdot 2^2 \cdot 5 \cdot 2 \cdot 3 \cdot 7 \cdot 2^3 \cdot 3^2 \cdot 2 \cdot 5 \cdot 11 \cdot 2^2 \cdot 3 \cdot 13 =$

$2^{10} \cdot 3^5 \cdot 5^2 \cdot 7^1 \cdot 11^1 \cdot 13^1$.

Therefore $p - q + r - s + t - u = 10 - 5 + 2 - 1 + 1 - 1 = 6$

19. Answer A: $\log_3 x = \log_3 x^2 + 1$

$\log_3 x = 2\log_3 x + 1$

$\log_3 x = -1$

$x = 3^{-1} = \frac{1}{3}$ $\quad y = \log_3 \frac{1}{3} = -1$

$(a, b) = \left(\frac{1}{3}, -1\right)$ Therefore $a + b = -\frac{2}{3}$.

20. Answer D: $\left(\left(\frac{1}{\log_2 3}\right)\left(\frac{1}{\log_3 2}\right)\left(\frac{1}{\log_3 4}\right)\left(\frac{1}{\log_2 9}\right)\right)^2 = \left(\left(\frac{\log 2}{\log 3}\right)\left(\frac{\log 3}{\log 2}\right)\left(\frac{1}{2\log_3 2}\right)\left(\frac{1}{2\log_2 3}\right)\right)^2 =$

$\left(\left(\frac{\log 3}{2\log 2}\right)\left(\frac{\log 2}{2\log 3}\right)\right)^2 = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$.

21. Answer B: $e^{x\ln 5} = 25$

$e^{\ln 5^x} = 25$

$5^x = 25$ and $x = 2$

Since the cardinality of a set is the number of elements in the set, the answer is 1.

22. Answer B: The units digit of 7^n where *n* is a whole number follows the pattern 1, 7, 9, 3, 1, 7, 9, 3, 1… Since the units digit repeats in identical cycles of 4 elements, the remainder of the division of the power *n* by 4 will give a power that will yield an equivalent result. Since $\frac{2015}{4} = 503$ with a remainder 3, $7^{2015} = 7^3$ which has a units digit of 3.

23. Answer C: $x = -\sqrt{5-\sqrt{5-\sqrt{5-\sqrt{5-\cdots}}}}$

$x = -\sqrt{5+x}$
$x^2 = 5 + x$
$x^2 - x - 5 = 0$

Using the quadratic formula $x = \frac{1\pm\sqrt{21}}{2}$. However, since x must be negative $x = \frac{1-\sqrt{21}}{2}$

24. Answer C: $\left(\frac{x^2}{4}+\frac{2}{x}\right)^{12} = \cdots + \binom{12}{7}\left(\frac{x^2}{4}\right)^5\left(\frac{2}{x}\right)^7 + \cdots$

$\binom{12}{7}\left(\frac{x^2}{4}\right)^5\left(\frac{2}{x}\right)^7 = \left(\frac{12!}{7!5!}\right)\left(\frac{x^{10}}{4^5}\right)\left(\frac{2^7}{x^7}\right) = \frac{12!2^7}{7!5!4^5}x^3 = 99x^3$

25. Answer B: $\log_{27} a + \log_9 b = \frac{7}{2}$ *and* $\log_{27} b + \log_9 a = \frac{2}{3}$

$\log_{27} a + \log_9 b + \log_{27} b + \log_9 a = \frac{7}{2} + \frac{2}{3}$

$\log_{27} ab + \log_9 ab = \frac{25}{6}$

$\frac{\log_3 ab}{\log_3 27} + \frac{\log_3 ab}{\log_3 9} = \frac{25}{6}$

$\frac{\log_3 ab}{3} + \frac{\log_3 ab}{2} = \frac{25}{6}$

$2\log_3 ab + 3\log_3 ab = 25$

$5\log_3 ab = 25$

$\log_3 ab = 5$

$ab = 3^5 = 243$

26. Answer D:

$l^2 = (2^5)^2 + (5(4)^3)^2 = 2^{10} + 25(4)^6 = 2^{10} + 25(2)^{12} = 2^{10}(1 + 25(2)^2) = 2^{10}(101).$

$l = 2^5\left(\sqrt{101}\right)$. Since $a = \sqrt{101}$ and $b = 5$ the sum $b + a$ is $5 + \sqrt{101}$.

27. Answer C: $8.5 = \frac{2}{3}\log\frac{E_1}{E_o}$ and $7.1 = \frac{2}{3}\log\frac{E_2}{E_o}$

$8.5 - 7.1 = \frac{2}{3}\log\frac{E_1}{E_o} - \frac{2}{3}\log\frac{E_2}{E_o}$

$1.4 = \frac{2}{3}\left(\log\frac{E_1}{E_o} - \log\frac{E_2}{E_o}\right) = \frac{2}{3}\left(\log\frac{\frac{E_1}{E_o}}{\frac{E_2}{E_o}}\right) = \frac{2}{3}\left(\log\frac{E_1}{E_2}\right)$

$\frac{3}{2}(1.4) = 2.1 = \log\frac{E_1}{E_2}$

$\frac{E_1}{E_2} = 10^{2.1}$. Therefore, the 8.5 earthquake must produce $10^{2.1}$ more times as much energy as the 7.1 earthquake.

28. Answer A: $\left(3x+\frac{1}{2x}\right)^3 = 4^3$

$(3x)^3 + 3(3x)^2\left(\frac{1}{2x}\right) + 3(3x)\left(\frac{1}{2x}\right)^2 + \left(\frac{1}{2x}\right)^3 = 64$

$27x^3 + 3(3x)\left(\frac{1}{2x}\right)\left(3x+\frac{1}{2x}\right) + \frac{1}{8x^3} = 64$

$27x^3 + 3(3x)\left(\frac{1}{2x}\right)(4) + \frac{1}{8x^3} = 64$

$27x^3 + 18 + \frac{1}{8x^3} = 64$

$27x^3 + \frac{1}{8x^3} = 46$

29. Answer A: $\log_x(xy^5) - \log_y\left(\frac{x^2}{\sqrt{y}}\right) = \log_x x + 5\log_x y - \left(2\log_y x - \frac{1}{2}\log_y y\right) =$
$\log_x x + 5\log_x y - 2\log_y x + \frac{1}{2}\log_y y$.
Since $\log_x y = -\frac{1}{4}$ then $\log_y x = -4$.
So $\log_x x + 5\log_x y - 2\log_y x + \frac{1}{2}\log_y y = 1 + 5\left(-\frac{1}{4}\right) - 2(-4) + \left(\frac{1}{2}\right)(1) = \frac{33}{4}$

30. Answer B: The inverse of $f(x) = 6\log_8(x-1) - 4$ has the form
$x = 6\log_8(y-1) - 4$. If its input is 2 then
$-2 = 6\log_8(y-1) - 4$
$2 = 6\log_8(y-1)$
$\frac{1}{3} = \log_8(y-1)$
$8^{\left(\frac{1}{3}\right)} = y - 1$
$2 = y - 1$
$y = 3$