1. **A** - Matrix multiplication is carried out from left to right. The product *CB* will yield a 3x3 matrix. That matrix product times *A* will yield a 3x4 matrix.

2. **B** - We calculate the determinant and set equal to 0. Thus, we obtain the equation $4y^2 - 10y = 0$. Factoring and solving for *y,* we obtain solutions of 0 and 5/2.

3. **C** - The determinant of a matrix product is equal to the product of the determinants. Determinant of *T* is 13 and the determinant of *Z* is 82. 13*82 = 1066.

4. **D** - Combine the first two equations and eliminate *y*. Do the same with the last two equations. Combine these resulting equations and eliminate either *x* or *z*. We can now back-substitute to solve for the remaining variables.

5. **B** - If we foil out the expression, remember that matrix multiplication is not necessarily commutative, thus *AT* does not equal *TA*. However, it is much easier to add *A* and *T*, then simply multiple by itself to obtain $\begin{bmatrix} 12 & 48 \\ 24 & 108 \end{bmatrix}$.

6. **B** - Augment *B* with the identity matrix and row-reduce *B* to the identity matrix, performing the same row operations to *I*. $[B \mid I] \rightarrow [I \mid B^{-1}]$.

7. **D** - The adjoint of a matrix is given by the transpose of the matrix of cofactors. We obtain the expression $\left((-4+0)(-8)\right)^2 - (-2 * -12)$ = **1000**.

8. **A** - Compute the characteristic polynomial $A - \lambda I = 0$. Matrix *A* happens to be singular, so we obtain $x^4 - 15x^3 = 0$. The solutions to this equation are the eigenvalues.

9. **D** - An eigenvector (*x*) of the matrix (*A*) will satisfy $Ax = \lambda x$. Thus, the easiest way to check is to multiply the answer choices on the left by *A* and check to see if the result is a scalar multiple of the initial eigenvector.

10. **D** - We row-reduce the matrix and count the number of pivot columns. Since there are two columns with leading 1's and zeroes below, the rank is **2**.

11. **B** - The product of the eigenvalues is given by the determinant of the matrix. Computing the determinant, we obtain **7**.

12. **A** - When a matrix is multiplied by a scalar, its determinant is multiplied by that scalar raised to the power of the number of rows of the matrix. Thus, the original determinant of *D* is given by $3^5|D| = 3402$, or $|D| = 14$. Thus, we compute $4^5|D| = 4^5 * 14 = 14336$.

13. **D** - We form matrix *A* as $\begin{bmatrix} 22 & 21 \\ 25 & 19 \end{bmatrix}$. Compute the determinant of this matrix to obtain **-107**.

14. **A** - Row-reduce *C* to obtain $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$. The entry in the 2nd row, 3rd column is **2**.

15. **B** - We proceed by multiplying the matrices $\begin{bmatrix} 5 & 6 \\ 3 & 1 \end{bmatrix}$ and $\begin{bmatrix} 6 & 12 \\ 6 & 8 \end{bmatrix}$ to obtain $\begin{bmatrix} \mathbf{66} & \mathbf{108} \\ \mathbf{24} & \mathbf{44} \end{bmatrix}$.

16. **D** - A matrix is idempotent if it is equal to its own square. Check each of the options.

17. **B** - A matrix is nilpotent if, when raised to higher and higher powers, the result tends towards the zero matrix. Choice B, when squared, yields the matrix of all zeroes.

18. **A** - The area of a triangle can be computed by taking half the absolute-value of the determinant of the coordinates, with a column of 1's added. Thus, compute $\begin{vmatrix} 5 & -7 & 1 \\ -2 & 3 & 1 \\ 4 & 9 & 1 \end{vmatrix}$ and divide by 2.

19. **D** - We seek to solve the inequality $33x + 12y \leq 415$. The largest possible value of *x* is 12. Tabulate the corresponding values of *y* and continue this until we get to the smallest value of *x* (1). Count up the ordered pairs *x*, *y* to obtain **195**.

20. **B** - The determinant of the inverse of a matrix is given by the reciprocal of the original matrix's determinant. Thus we have $|3N| = 16$, and $|(3N)^{-1}| = \mathbf{\frac{1}{16}}$.

21. **D** - The determinant of *D* must be non-negative, since the product of the determinants is equal to the determinant of the product. In other words, since $C * C = D$, $|C||C| = |D|$. Only choice **D** has a non-negative determinant.

22. **C** – The largest possible value of the determinant is 40, and the smallest positive value is 8. Thus, the ratio is 1 to 5.

23. **A** – We multiply $\begin{bmatrix} -5 & 4 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 6 \\ -4 \end{bmatrix}$ to obtain the new point $(\mathbf{-46, 10})$.

24. **D** – Calculate the slope-interecept form of the line containing the first two points. We obtain $y = \frac{1}{3}x + \frac{25}{3}$. Plug in 12 to obtain the solution.

25. **A** – All properties are true. The matrix is equal to its transpose and conjugate transpose. It is clearly square, as well as singular, since its entries are all identical.

26. **D** – We can identify the original system of linear equations (*Ax* = *b*) by looking at the matrix in the denominator to discover the coefficient matrix (*A*) for the variables. The matrix in the numerator is the same, however, the second column, denoting the variable *y*, has the solution matrix *b* substituted instead. Use substitution and elimination to solve the resulting system of linear equations. We find *x* = 1 and *z* = -1, thus *x* – 3*z* = **4**.

27. **C** - We construct matrix *A* as $\begin{bmatrix} -1 & -5 & -9 \\ 2 & -2 & -6 \\ 5 & 1 & -3 \end{bmatrix}$. The sum of its entries is **-18**.

28. **A** – The trace is computed as the sum of the entries on the main diagonal. Thus, the trace is $3x^2 - 12x + 16$. Setting this equal to 7 yields the equation $3x^2 - 12x + 9 = 0$, which factors to $(3x - 3)(x - 3)$. Solving for the zeroes of this equation gives *x* = **1, 3**.

29. **C** – The columns are the integers 8,9, …, 91 which includes 84 integers. The rows are the multiples of these integers from 1 to 56, thus there are 56 rows. The dimension is $\mathbf{56 \times 84}$.

30. **C** – The smallest I was able to construct is as follows $\begin{vmatrix} 5 & 7 & 1 \\ 9 & 2 & 4 \\ 3 & 6 & 8 \end{vmatrix} = -412$. If we compute the determinant $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$, we obtain $aei + bfg + cdh - ceg - bdi - afh$. If we maximize the values *b*, *d*, *i*, choosing 7, 8, and 9 for these, we then need to minimize *ch*, *ae*, and *fg*. We pick 1, 2, and 3 for *c*, *e*, and *g*. This leaves 4, 5, and 6 for *a*, *f*, and *h*.