Practice round: The trinomial factors into (9*x* +8)(4*x* + 3); the value of A + B + C + D = **24.**

1. **Product of the positive roots is 51.**

- $|2x+5| = x-7$
  Solve in parts: 2x + 5 = x-7 and 2x + 5 = -x + 7. The solutions to these equations appear to be -12 and 2/3, but both are extraneous and do not satisfy the original equation, so **there are no positive roots to this equation.**
- Solve by factoring: (5x + 8)(x – 3) = 0. So x = -8/5 and 3. **Positive root is 3.**
- $10-5(2x+3)-4(x-2) = -2[3(x-2)-(x-6)]-7$ By careful distributing and combining terms, we solve and get -14x + 3 = -6x + 12 + 2x -12 – 7, which gives us -10x = -10, so **x = 1.**
- Solve by factoring, recognizing that 51 is divisible by 3: x(x+3)(x-17)= 0. The roots are -3, 0, and 17. **The positive root is 17.**

2. **The sum of these solutions is $\frac{1409}{64}$.**

- $(x-2)^2-5(x-2)+6=0$ Start by temporarily letting a = (x-2) and factor as a^2 -5a + 6 = 0. This gives us a = 2 and a = 3. Then substitute (x-2) back in place of a: the solutions are **x = 4 and x = 5.**
- $(\log_{49} x)(\log_2 7) = -3$ Using change of base property, we get $\frac{\log x}{\log 49}\cdot\frac{\log 7}{\log 2} = -3$ which simplifies to $\frac{\log x}{2\log 2} = -3$ and then $\log_4 x = -3$, so **x = 1/64.**
- $x^{-1} = 2^{-1} - 6^{-1}$ This equation is simply $\frac{1}{x} = \frac{1}{2} - \frac{1}{6}$, so **x = 3.**
- $x - \sqrt{x+6} = 6$. Start by isolating the radical, and then square both sides of the equation: $x-6 = \sqrt{x+6} \Rightarrow x^2 - 12x + 36 = x+6$. This gives us $x^2 - 13x + 30 = 0$, which factors and solves to give us x = 3 and 10. Checking in the original equation, we find the only valid root is **10.**

3. The value of A^B = **1/6.**

A: To solve $3 \le |2x-3| < 7$, we separate it into the parts: |2x-3| ≥ 3 and |2x-3| <7, which become (2x-3 ≥ 3 OR 2x-3 ≤ -3) and (2x-3 < 7 AND 2x-3 > -7). These solve out to give us (x ≥ 3 or x ≤ 0) AND (x < 5 and x > -2). The final solution is {x: -2 < x ≤ 0 or 3 ≤ x < 5} which includes these integers: -1, 0, 3, and 4. **Their sum is 6.**

B: Solving the disjunction $-3(2-3x) < -6\left(\frac{5}{3}-\frac{x}{2}\right)$ *or* $-5(x+3) > -8\left(\frac{x}{2}+\frac{1}{4}\right)+x$,

the first inequality gives us x < -2/3; the second x < -13/2. The union of these is x < -2/3, and the largest integer in this set is **-1.**

4. **3**

A**:** $i^{2015} = -i$ $i^{-99} = i$ $(1-i)^{11} = -32-32i$ $|5-12i| = 13$; the expression equals -45-32*i*

B: The reciprocal of (3 – 4*i*) = $\frac{3+4i}{25}$; the square of $\frac{2+7i}{5i}$ = $\frac{45+28i}{25}$. Their sum equals $\frac{48+32i}{25}$.

C: $\sqrt{-15} = i\sqrt{15}$; $\sqrt{-60} = 2i\sqrt{15}$; $(i\sqrt{5})^2 = -5$. The expression equals -30 - -5 = -25

5. A + B + C = 56.

A: The length of the major axis is 12, that of the minor axis is 32; the SUM is **44**

B: The vertex is (5, -7), the parabola opens to the left, and p = -2, so the focus is (3, -7), the sum is **-4.**

C = The equation can be changed to the form: $\frac{(x-4)^2}{9}+\frac{(y+3)^2}{25}=1$; the endpoints of the minor axis are (1, -3) and (7, -3) and those of the major axis are (4, 2) and (4, -8). The sum of the abscissas is **16.**

6. AB + C + D = 1,115

A: $\frac{32}{125}=\frac{625}{4}(r)^7$, which simplifies to $r^7=\frac{125}{32}\cdot\frac{625}{4}$, so $r^7=\left(\frac{2}{5}\right)^7$ and $r=\left(\frac{2}{5}\right)$.

$a_6=\frac{625}{4}\left(\frac{2}{5}\right)^5$. **$a_6$=8/5**

B: First we must find the common difference d: 15 = 7 + d(6), which gives us d = 4/3. Then we can solve for n: 139 = 7 + (4/3)(n-1), which gives us n = **100.**

C: First solve for the common difference: -49 = -28 + d(7) which tells us d = -3. Then find the 4th term: -28 = a_4 + (-3)(5), which gives us a_4 = **-13.**

D: First, find ***r*:** $8=216\cdot r^7$, so $r=\frac{1}{3}$. We can then solve for a_1; a_1 = 648. Then using the sum formula: $S_5=\frac{648-8\left(1/3\right)}{1-\left(1/3\right)}$ and simplifying, we get ***S_5* = 968.**

7. The true statements are: **A, D, E, H, J.**

The function, when factored, is $f(x)=\frac{2x(x+6)(x-1)(2x-3)(2x+3)}{(2x-3)(x+6)(x+2)(x-1)}$, which reduces to the function $f_r(x)=\frac{2x(2x+3)}{(x+2)}$. The vertical asymptote is *x* = -2, there are no horizontal asymptotes, the slant asymptote is *y* = 4*x* -2, and removable discontinuities occur at *x* = -6, 1, and 3/2: (-6, -27), (1, 10/3), and (3/2, 36/7).

8. ***B – ACD* = 4,047**

A: This is a geometric series; using formula $S_n=\frac{a_1-a_n\cdot r}{1-r}$, we get $S_6=\frac{-2-64\cdot(-2)}{1-(-2)}=42$

B: This is an arithmetic series; using the formula $S_n=\frac{n}{2}(a_1+a_n)$, we get $S_{101}=\frac{101}{2}(-3+97)=4747$

C: This is an infinite geometric series; using the formula $S=\frac{a_1}{1-r}$, we get $S=\frac{5/2}{1-2/5}=\frac{25}{6}$

D. Again, this is a geometric series. Using the formula $S_n=\frac{a_1-a_n\cdot r}{1-r}$, we have $13{,}120=\frac{a_1(1-3^8)}{1-3}$, which gives us -26,240 = a_1(1-6561), which leads to a_1 = 4.

9. $\mathbf{14\sqrt{2}-2}$ Using the complex conjugates theorem, you know that both $-1-\sqrt{5}$ and $-1+\sqrt{5}$ are roots. Using the sum and product of roots of a quadratic, we see that they correspond to the quadratic factor x^2+2x-4. By long division, the other factor of the quartic polynomial is x^2-4x+2. This can be solved by the quadratic formula to give us the roots $2+\sqrt{2}$ and $2-\sqrt{2}$. Comparing

these three roots to find the smallest, we focus on $2-\sqrt{2}$ and $-1+\sqrt{5}$. $2-\sqrt{2}$ is approximated by 2 – 1.4, so it is close to 0.6. $-1+\sqrt{5}$ is approximated by -1 + 2.2, so it is close to 1.2. We can conclude that $2-\sqrt{2}$ is the smallest of the three roots. The given expression $\sqrt{338}=13\sqrt{2}$, so we have $13\sqrt{2}-(2-\sqrt{2})$ = $\mathbf{14\sqrt{2}-2}$

10. **81/10**

A: The equation transforms to $\frac{(x-4)^2}{9}+\frac{(y+1)^2}{25}=1$, which tells us a = 5, b = 3, and area therefore equals 15π.
B: The y-intercept is 4 and the x-interept is -6, so the area of this triangular region is 12.
C: The area of the sector is a proportional part of the area of the entire circle: $\frac{135}{360}(144\pi)=54\pi$
D: The two bases are 4 and 2. Dropping altitudes from the upper vertices to the longer base we have two 30-60-90 triangles formed on the left and right sides of the trapezoid. The altitude of each triangle equals the height of the trapezoid, which is $\sqrt{3}$. The area is $3\sqrt{3}$.

11. $\mathbf{120\sqrt[3]{12}}$

A: This simplifies to $3\sqrt[3]{12}+\frac{\sqrt[3]{12}}{2}-6\sqrt[3]{12}$ which equals $-\frac{5\sqrt[3]{12}}{2}$.
B: This simplifies to $\frac{\sqrt{3}}{3}+6\sqrt{3}-\frac{7\sqrt{3}}{3}$, which equals $4\sqrt{3}$.
C: This is the factored form for the difference of two cubes, so the product equals (2 – 5) or -3.
D: Converting to exponential form is efficient: $\frac{\sqrt[6]{2^{12}}}{\sqrt[12]{3^6}}=\frac{2^2}{\sqrt{3}}=\frac{4\sqrt{3}}{3}$.

12. **600**
A: The equation to represent this situation is: .06(700) = .05x + .12(700 – x), which solves to x = 600.
B: The rate of filling is ½ the pool per hour, the rate of emptying is 1/10 of the pool per hour. The equation to represent filling an empty pool in x hours is: $\frac{1}{2}x-\frac{1}{10}x=1$, which gives us x = 2½ hr.
C: Let x = time spent walking; then (1½ - x) = time spent biking. The equation that indicates the distance walked equals the distance biked is 2x = 10(1½ - x); this solves to x = 5/4, so the distance to the shop is 2½ miles.

13. **15**
A: The abscissa of the midpoint will be the average of the x-coordinates, which is 2.
B: First find the slope of the segment with the given endpoints: m = -1. The perpendicular line will have its slope as the opposite reciprocal of this, which is 1. The bisector passes through the midpoint of the segment, which is the point (-1, -4). Solving for the y-intercept of the line with slope of 1 and passing through the point (-1.-4): -4 = 1(-1) +b, so b = -3.
C: The vertex occurs where |x+2| = 0, which gives us y = 5.

14. **-2**
The factored form of the trinomial is (6x - 25)(9x + 8).