Q-0

$A \to 3x < 12 \to x < 4\, and\, 2x \geq 4 \to x \geq 2 \to 2 \leq x < 4 \to 2+3=5$

$B \to \frac{4-3}{0--2} = \frac{n-4}{6-0} \to \frac{1}{2} = \frac{n-4}{6} \to 6 = 2n-8 \to n=7$

$C \to 2x \leq -2 \to x \leq -1\, or\, 4x \leq -8 \to x \leq -2 \to greatest\, \text{int}\, eger\, -1$

$D \to 2^4 = 16$

$5+7-1+16=27$

Q-1

$\left(\frac{-3+5}{2}, \frac{4-2}{2}\right) = (1,1)$

- Get perpendicular bisector, need slope and midpoint: $m = \frac{4--2}{-3-5} = \frac{-3}{4} \to \frac{4}{3}$

$4x-3y=c \to 4x-3y=1$

$2016 = 2^5 \bullet 7 \bullet 3^2 \to sum = \left(2^0+2^1+2^2+2^3+2^4+2^5\right)\left(7^0+7^1\right)\left(3^0+3^1+3^2\right) = 63(8)(13) = 6552$

$i^0 = 1$

$\frac{2L-5--2}{7-3} = \frac{2L-3}{4}$

$\frac{L+4-5}{-6--9} = \frac{L-1}{3} = \frac{2-3}{4} \to 4L-4 = 6L-9 \to L = \frac{5}{2}$

$\sqrt{(U-3)^2+(8--4)^2} = 13 \to U^2-6U+9+144=169$

$U^2-6U-16 = (U-8)(U+2) \to 8-2=6$

$4-3+1+\frac{5}{2}+6 = 10.5\, or\, \frac{21}{2}$

Q2-Draw apicture!! Since the vertex is 5 units below focus the directrix will be 5 units below vertex and therefore y= -3

Transform equation: $\frac{y^2}{16} - \frac{x^2}{4} = 1 \to a = 4\, and\, b = 2$: The slopes are plus and minus a/b so the product of the slopes is 2 times -2 which is -4

$$25\left(x^2-8x+16\right)+9\left(y^2+2y+1\right)=-184+400+9$$

Transform: $\dfrac{(x-4)^2}{9}+\dfrac{(y+1)^2}{25}=1 \rightarrow ctr\,(4,-1) \rightarrow \ foci\,(4,4)\, and\, (4,-6)$

$$4-6=-2$$

-3-4-2=-9

Q3-Set Discriminant to be less than zero:

$$64-4(2n-1)<0 \rightarrow 64-8n+4<0 \rightarrow 8n>68 \rightarrow n>8.5 \rightarrow 9$$

$x^3-8=0$: Sum of roots formula –b/a gives the sum of the roots as zero. Since 2 is a real root the other 2 imaginary roots must sum to -2

$$C-a^2+2abi-b^2=7-24i \rightarrow a^2-b^2=7 \rightarrow 2ab=-24$$

$$4-3i \rightarrow |a+bi|=\sqrt{4^2+(-3)^2}=5$$

$$D-\frac{(13+i)(1-2i)}{(1+2i)(1-2i)}=\frac{15-25i}{5}=3-5i \rightarrow -2$$

9-2+5-2=10

Q4-. Draw yourself a picture: $b+h=31 \rightarrow b=31-h \rightarrow 2x+b=50 \rightarrow x=25-\dfrac{b}{2} \rightarrow h^2=x^2-\left(\dfrac{b}{2}\right)^2$

$$h^2=\left(25-\frac{b}{2}\right)^2-\left(\frac{b}{2}\right)^2=625-25b=625-25(31-h)$$

$$h^2-25h+150=0 \rightarrow (h-10)(h-15)=0 \rightarrow 25$$

Draw a picture: The side of the octagon can be expressed as either 2x or x root two. Set them equal and

$$x\sqrt{2}=10-2x \rightarrow 2x+x\sqrt{2}=10 \rightarrow x\left(2+\sqrt{2}\right)=10$$

solve for x:

$$x=\frac{10}{2+\sqrt{2}} \bullet \frac{2-\sqrt{2}}{2-\sqrt{2}}=\frac{10\left(2-\sqrt{2}\right)}{2}=10-5\sqrt{2}$$

Multiple x by root 2 and you have the answer: $\left(10-5\sqrt{2}\right)\sqrt{2}=10\sqrt{2}-10$

$$10\sqrt{2}-10+25=10\sqrt{2}+15$$

Q-5

Draw a 6 by 8 and you will quickly see what products work. All factors that have 3 and 6 will work. There are 24 such factors out of 48 so 1/2

Greater than zero means two negatives or two positives so:

$$\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)=\frac{4}{9}$$

$$\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)=\frac{1}{9}$$

$$\frac{5}{9}+\frac{1}{2}=\frac{10+9}{18}=\frac{19}{18}$$

Q-6 $\dfrac{\left(5^{2013}\right)^2\left[\left(5^3\right)^2-5^2\right]}{\left(5^{2013}\right)^2\left[\left(5^2\right)^2-1\right]}=\dfrac{\left(5^3-5\right)\left(5^3+5\right)}{\left(5^2-1\right)\left(5^2+1\right)}=\dfrac{130\bullet 120}{26\bullet 24}=25$

Everything cancels to zero except $i^0=1$

Let X=2016, then:

$$\sqrt{(x-4)(x-2)(x+4)(x+2)+36}=\sqrt{\left(x^2-16\right)\left(x^2-4\right)+36}=\sqrt{x^4-20x^2+100}=\sqrt{\left(x^2-10\right)^2}=\left(x^2-10\right)$$

25+1+2016+10=2052

Q-7 $\dfrac{x}{3}+\dfrac{x}{4}+\dfrac{x}{5}+52=x\rightarrow\dfrac{47}{60}x+52=x\rightarrow x=240$

8 steps left and 10 steps down: 2-8/5=2/5 and 7-2=5 product of these is 2

2013, 2014, 2015, 2012, 2017, 2010, 2019, 2008. The even numbered terms are 2014, 2012, 2010, 2008,... since we are looking for the 2016th term we can just look for the 1008 term of the even positioned terms. They are arithmetic with a common difference of -2. 2014 +1007(-2)=0

240+2+0=242

Q-8-A cube has 6 faces and each would have an area of 4, so the edge would be 2. The diameter of the inner sphere is the space diagonal for the cube, so:

$$3L^2 = 2^2 \rightarrow L^2 = \frac{4}{3} \rightarrow 6\left(\frac{4}{3}\right) = 8$$

The surface area of the top pyramid to the bottom pyramid is 1:2. Therefore the scale factor is:

$$\frac{\sqrt{2}}{2} = \frac{h}{h+2} \rightarrow 2h = h\sqrt{2} + 2\sqrt{2} \rightarrow h\sqrt{2} - 2h = -2\sqrt{2} \rightarrow h\left(\sqrt{2}-2\right) = -2\sqrt{2}$$

$$h = \frac{-2\sqrt{2}}{\left(\sqrt{2}-2\right)} \bullet \frac{\sqrt{2}+2}{\sqrt{2}+2} = \frac{-4-4\sqrt{2}}{-2} = 2+2\sqrt{2} \rightarrow 2+2\sqrt{2}+2 = 4+2\sqrt{2}$$

$$8+4+2\sqrt{2} = 12+2\sqrt{2}$$

Q-9

$$\frac{16}{3}\left(c+\frac{1}{5}\right) = \frac{4}{3}+4c \rightarrow 16c+\frac{16}{5} = 4+12c$$

$$4c = \frac{4}{5} \rightarrow c = \frac{1}{5} \rightarrow \frac{1}{5}+\frac{1}{5} = \frac{2}{5} = 40\%$$

C	A	T		C	A	T
0	10	0		1/7	14	2
1	2	2		1/7	-2	-2/7
1/6	12	2		1/7	12	12/7

$$\frac{1}{7} \div \frac{1}{6} = \frac{6}{7}$$

$$40 \bullet \frac{6}{7} = \frac{240}{7}$$

Q-10

$$\frac{24}{2} - \frac{24}{6} = 12 - 4 = 8$$

$$\frac{b--3}{a-5} = -2 \rightarrow b+3 = -2(a-5) \rightarrow b+2a = 7$$

$$\frac{5-b}{6-a} = \frac{1}{2} \rightarrow 2(5-b) = 6-a \rightarrow a-2b = -4$$

a=2 and b=3 a+b=5

You can divide out $x^2+5x-24$ as long as you know you are throwing away roots 3 and -8

$x^2-7x+12 = (x-3)(x-4) \rightarrow 3,4,3,-8$ 4--8=12

$$8 \bullet 5 \bullet 12 = 480$$

Q-11

$$9^{k-1}=3^{2k-2}=\frac{3^{2k}}{9}=\frac{(12)^2}{9}=16$$

$$2\log_3(x+4)-\log_3(4x-11)-2=0$$

$$\log_3\left(\frac{(x+4)^2}{4x-11}\right)=2\rightarrow 9=\frac{x^2+8x+16}{4x-11}$$

$$36x-99=x^2+8x+16\rightarrow x^2-28x+115=0\rightarrow 115$$

$$\frac{2^{\log_4 108}}{2^{\log_4 3}}=\frac{2^{\log_2\sqrt{108}}}{2^{\log_2\sqrt{3}}}=\sqrt{\frac{108}{3}}=6$$

$$\frac{4}{\log_{10}5}-\frac{2}{\log_4 5}=4\log_5 10-2\log_5 4=\log_5\frac{10^4}{4^2}$$

$$\log_5 625=4$$

(115+16)(6+4)=1310

Q-12-Draw a picture and you see that we get 2 similar triangles:

$$\frac{3}{4}=\frac{8}{y}\rightarrow=\frac{32}{3}$$

$$4+\frac{32}{3}=\frac{44}{3}$$

Draw a picture and call the side of the triangle 2S. We then get:

$$6S=\pi\left(\frac{2S}{\sqrt{3}}\right)^2=\frac{4S^2\pi}{3}\rightarrow\frac{18}{4\pi}=S=\frac{9}{2\pi}$$

$$\frac{9}{2\pi}\bullet\frac{4}{\sqrt{3}}=\frac{36\sqrt{3}}{6\pi}=\frac{6\sqrt{3}}{\pi}$$

$$(2x)^2+(3x)^2=16\bullet 13=13x^2\rightarrow x=4\rightarrow 4\bullet 4=16$$

$$\frac{6\sqrt{3}}{\pi}\bullet\frac{44}{3}\div 16=\frac{11\sqrt{3}}{2\pi}$$

Q-13 You will need some formulas here. $\pi rl=\frac{3}{5}\left(\pi rl+\pi r^2\right)\rightarrow l=\frac{3}{5}(l-r)\rightarrow\frac{2}{5}l=\frac{3}{5}r\rightarrow\frac{r}{l}=\frac{2}{3}$

The hexagonal pyramid creates a 6-8-10 right triangle where the hypotenuse of the right triangle is the slant height of the cone and the base edge of 6 is the radius. $\pi rl=\pi(6)(10)=60\pi$

MU= $\frac{2}{3} \bullet 60\pi = 40\pi$

Q-14 $3 \bullet 2 \bullet 1 \bullet 5 \bullet 4 \bullet 3 \bullet 2 \bullet 1 = 720$

${}_{10}C_3 - {}_6C_3 = 120 - 20 = 100$

720 + 100 = 820

Answers;

0- 27
1- 10.5 or $\frac{21}{2}$
2- -9
3- 10
4- $10\sqrt{2}+15$
5- $\frac{19}{18}$
6- 2052
7- 242
8- $12+2\sqrt{2}$
9- $\frac{240}{7}=34\frac{2}{7}$
10- 480
11- 1310
12- $\frac{11\sqrt{3}}{2\pi}$
13- 40π
14- 820